

STATISTICAL PROPERTIES OF  
OCEAN WAVE GROUPS

Arthur Ronald Nelson



# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

STATISTICAL PROPERTIES OF OCEAN WAVE GROUPS

by

Arthur Ronald Nelson

March 1980

Thesis Advisor:

W. C. Thompson

Approved for public release; distribution unlimited.

T192009



REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Statistical Properties of Ocean Wave Groups		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; March 1980
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Arthur Ronald Nelson		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE March 1980
		13. NUMBER OF PAGES 133
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Ocean wave groups Wave group statistics		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  Individual wave groups were identified by an analysis of digitized wave records. The analysis technique is based on a comparison between short-term variance and wave record variance. Once wave groups were identified, wave parameters of interest were then derived. Analysis was performed on 338 wave records obtained from a Waverider buoy covering a wide range of height, period, and steepness. Statistical relationships		



between the parameters obtained were determined for all wave groups, and for selected extreme wave groups. It was found that as the relative energy of the group increases the number of waves per group increases, and the average group period approaches the spectral peak period. Wave steepness was shown, however, not to depend upon the group energy. Groups from low steepness wave records tend to contain larger numbers of waves. It was also shown that the height of the single highest wave in a group tends to approximate the significant height of the wave record.





Approved for public release; distribution unlimited.

Statistical Properties of Ocean Wave Groups

by

Arthur Ronald Nelson  
Lieutenant, United States Navy  
B.S., University of Utah, 1974

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN METEOROLOGY AND OCEANOGRAPHY

from the  
NAVAL POSTGRADUATE SCHOOL  
March 1980



## ABSTRACT

Individual wave groups were identified by an analysis of digitized wave records. The analysis technique is based on a comparison between short-term variance and wave record variance. Once wave groups were identified, wave parameters of interest were then derived. Analysis was performed on 338 wave records obtained from a Waverider buoy covering a wide range of height, period, and steepness. Statistical relationships between the parameters obtained were determined for all wave groups, and for selected extreme wave groups. It was found that as the relative energy of the group increases the number of waves per group increases, and the average group period approaches the spectral peak period. Wave steepness was shown, however, not to depend upon the group energy. Groups from low steepness wave records tend to contain larger numbers of waves. It was also shown that the height of the single highest wave in a group tends to approximate the significant height of the wave record.



# TABLE OF CONTENTS

I.	INTRODUCTION - - - - -	11
II.	WAVE GROUP DETERMINATION - - - - -	14
	A. WAVE GROUP CONSIDERATIONS - - - - -	14
	B. WAVE GROUP DEFINITIONS - - - - -	16
III.	WAVE DATA ACQUISITION, SELECTION AND ANALYSIS - - - - -	21
IV.	WAVE RECORD AND WAVE GROUP PARAMETERS - - - - -	24
	A. WAVE RECORD PARAMETERS - - - - -	24
	1. Spectral Peak Period: $T_R$ - - - - -	24
	2. Record Variance: $V_R$ - - - - -	24
	3. Significant Height: $H_R$ - - - - -	25
	4. Wave steepness: $G_R$ - - - - -	25
	5. Record Group Duration: $D_R$ - - - - -	26
	B. WAVE GROUP PARAMETERS - - - - -	26
	1. Wave Group Duration: $D$ - - - - -	26
	2. Number of Waves per Group: $N_G$ - - - - -	27
	3. Wave Group Period: $T_G$ - - - - -	27
	4. Average Wave Group Variance: $V_G$ - - - - -	27
	5. Highest Individual Wave Height per Group: $H_M$ - - - - -	27
	C. WAVE GROUP TO WAVE RECORD PARAMETERS - - - - -	28
	1. Wave Group Variance to Wave Record Variance: $V_G/V_R$ - - - - -	28
	2. Wave Group Period to Spectral Peak Period: $T_G/T_R$ - - - - -	29
	3. Highest Group Wave Height to Significant Wave Height: $H_M/H_R$ - - - - -	29



V.	RESULTS AND INTERPRETATION	- - - - -	30
A.	ANALYSIS SCHEDULE	- - - - -	30
B.	ALL WAVE GROUPS	- - - - -	32
	1. Frequency Distributions (graphs)	- - - -	32
	a. $N_G$	- - - - -	32
	b. $T_G/T_R$	- - - - -	35
	c. $V_G/V_R$	- - - - -	38
	d. $H_M/H_R$	- - - - -	40
	e. $D_R$	- - - - -	41
	2. Statistical Measures (tables)	- - - - -	41
C.	EXTREME WAVE GROUPS	- - - - -	42
	1. Frequency Distribution (graphs)	- - - -	43
	a. Largest $N_G$ per Wave Record	- - - - -	43
	b. Largest $V_G/V_R$ per Wave Record	- - - -	45
	2. Statistical Measures (tables)	- - - - -	46
VI.	SUMMARY	- - - - -	47
	TABLES	- - - - -	50
	FIGURES	- - - - -	57
	APPENDIX	- - - - -	113
	LIST OF REFERENCES	- - - - -	129
	INITIAL DISTRIBUTION LIST	- - - - -	131





## LIST OF TABLES

I.	Definitions of wave steepness and wave age - - - - -	50
II.	Parameter matrix for all wave groups - - - - -	51
III.	Parameter matrix for extreme wave groups - - - - -	52
IVA-E.	Statistical data for all wave groups - - - - -	53
VA-B.	Statistical data for extreme wave groups - - - - -	56



## LIST OF FIGURES

1.	Illustration of zero upcrossing method definitions - - - - -	57
2.	Wave record and group waveform analysis for wave record number 143 - - - - -	58
3.	Wave record and group waveform analysis for wave record number 285 - - - - -	59
4.	Illustration of wave group boundary definition - - - - -	60
5.	Location of Monterey Bay Waverider buoy - - - - -	61
6.	CEDN graphical spectra example - - - - -	62
7.	CEDN tabular spectra example - - - - -	63
8.	Spectral analysis of wave record number 143 - - - - -	64
9.	Spectral analysis of wave record number 285 - - - - -	65
10.	Height, period, and steepness distribution of wave records sampled - - - - -	66
11.	Distribution of $N_G$ for all groups - - - - -	67
12.	Distribution of $N_G$ for $T_G/T_R$ bands - - - - -	69
13.	Distribution of $N_G$ for $V_G/V_R$ bands - - - - -	71
14.	Distribution of $N_G$ for $H_M/H_R$ bands - - - - -	73
15.	Distribution of $N_G$ for $G_R$ bands - - - - -	75
16.	Distribution of $T_G/T_R$ for all groups - - - - -	77
17.	Distribution of $T_G/T_R$ for $V_G/V_R$ bands - - - - -	79
18.	Distribution of $T_G/T_R$ for $H_M/H_R$ bands - - - - -	81
19.	Distribution of $T_G/T_R$ for $G_R$ bands - - - - -	83
20.	Distribution of $V_G/V_R$ for all groups - - - - -	85



21.	Distribution of $V_G/V_R$ for $H_M/H_R$ bands - - - - -	87
22.	Distribution of $V_G/V_R$ for $G_R$ bands - - - - -	89
23.	Distribution of $H_M/H_R$ for all groups - - - - -	91
24.	Distribution of $H_M/H_R$ for $G_R$ bands - - - - -	93
25.	Distribution of $D_R$ for all groups - - - - -	95
26.	Distribution of $N_G$ for extreme groups - largest $N_G$ - - - - -	97
27.	Distribution of $T_G/T_R$ for extreme groups - largest $N_G$ - - - - -	99
28.	Distribution of $V_G/V_R$ for extreme groups - largest $N_G$ - - - - -	101
29.	Distribution of $H_M/H_R$ for extreme groups - largest $N_G$ - - - - -	103
30.	Distribution of $N_G$ for extreme groups - largest $V_G/V_R$ - - - - -	105
31.	Distribution of $T_G/T_R$ for extreme groups - largest $V_G/V_R$ - - - - -	107
32.	Distribution of $V_G/V_R$ for extreme groups - largest $V_G/V_R$ - - - - -	109
33.	Distribution of $H_M/H_R$ for extreme groups - largest $V_G/V_R$ - - - - -	111



## ACKNOWLEDGEMENTS

I wish to express my sincere thanks to Professor Warren C. Thompson for many of the ideas, concepts, and methods used in this thesis. In particular, the credit for the procedures relating to the relationships between wave groups and individual waves in a wave record and the frequency spectrum of the wave record goes to Professor Thompson. His experience, counsel, and sacrifice of time were invaluable in both the research and writing of this thesis. He is a gentleman and a scholar in every sense.

I also wish to thank and recognize the efforts of the night and weekend computer operators in the W. R. Church Computer Center. Their help, and courteous and timely return of products helped to make the whole thesis research effort rewarding.

Finally, I wish to thank my wife, Janice, for the patience, understanding, and support that she showed, not only during the thesis quarters, but throughout our two year assignment at the Naval Postgraduate School. She truly made this educational experience a worthwhile endeavor.





## I. INTRODUCTION

A visual observation of the sea surface reveals that it is a highly complex and irregular surface. The fact that most waves have no apparent relation to or dependence on each other has prompted most studies or analyses of the sea surface to be statistical in nature. This type of approach has been necessary for lack of a better one, but it has also been highly successful in enabling the mechanisms of ocean waves to be understood.

Visual observation also shows, however, that ocean waves commonly appear in packets or groups. These waves are quasi-periodic and although the wave heights are not equal they have been shown to be dependent on each other (Rye, 1974). The existence of wave groups has been known to sea-faring men for many years. Out of their experiences have come folk lore including the saying that "every seventh wave is the largest," and an old Icelandic saying that "large waves rarely come alone." The fact that the phenomenon of wave grouping has been known and respected by sailors for ages makes it all the more surprising that it is only in recent years that wave groups have been systematically studied.

The economic impact of wave groups is only now being realized. It is recognized that groups of waves, not



necessarily large, may be damaging. Some of the engineering problems that have been found to be related to wave groups are the stability of rubble-mound breakwaters, the slow drift oscillations of moored floating platforms, the stability of ships underway, and induced seiching in harbors or bays that may cause damage to moored craft and in extreme conditions may be responsible for local flooding. Design approaches today do not generally take wave group characteristics into account.

Two approaches have been conventionally employed in the analysis of wave groups. The first involves using a statistical theory of runs while the second consists of examining the statistics of wave envelopes in relation to power spectra (Goda, 1976). Among those working on the distribution of the lengths of runs with simulated or actual wave data have been Goda (1970), Wilson and Baird (1972), Rye (1974), and Siefert (1976). Ewing (1973) and Chou (1978) have worked with the application of wave envelope statistics to wave groups. Mollo-Christensen and Ramamonjiarisoa (1978) recently proposed a non-linear model for wind waves in which "envelope solitons" or groups of Stokes waves propagate at the same speed for all frequencies rather than obeying a linear frequency dispersion relation.

A common thread running through all of these studies is that they employ power spectra generated by Fourier transform techniques. Non-spectral analyses have been



relatively few in number probably owing to the success that spectral techniques and methods have enjoyed. Hamilton, Hui, and Donelan (1979) have recently proposed a simple non-spectral statistical model with decoupled wave groups that improves spectral estimates of real or laboratory wind waves. Thompson (1972), Smith (1974), and more recently Sedivy (1978) have devised a non-spectral technique for identifying wave groups and then generating wave group statistics of interest.

This study utilizes the method used by Sedivy (1978) whereby a computer program analyzes digitized wave records by means of an rms smoothing filter. This filter employs a running short-term variance computation. The resulting smoothed record is then the basis for identifying individual wave groups from which various wave-group parameters of interest are obtained. Relationships are examined between wave-group parameters, and between wave group and wave record parameters. Possible relationships between power spectra and wave-group and wave-record parameters are examined. The relation between individual waves and the power spectrum is also examined.

It is hoped that the statistics generated in this study will be of use to the engineer, and when looked at in conjunction with the results of others will help provide the basis for theories on ocean wave groups.



## II. WAVE GROUP DETERMINATION

### A. WAVE GROUP CONSIDERATIONS

As mentioned, the most popular approach at present for waveform or wave group analysis is to work in the frequency domain, or in other words, with power spectra. The basis of this type of analysis has its roots in the assumption that ocean waves are composed of a linear summation of an infinite number of simple sinusoidal waves of infinitesimal amplitudes and random phases. Ocean waves can then be treated as a normal or Gaussian process. Longuet-Higgins (1952) showed that ocean waves of narrow-banded spectra have a distribution of wave heights that are approximated by a Rayleigh distribution. Goda (1974), and Chakrabarti and Cooley (1977) have shown that the Rayleigh distribution holds or at least is a good approximation even if the process is not narrow banded. Other investigators, Forristall (1978), disagree and have put forward evidence to show that the Rayleigh distribution is not a good approximation for wide banded processes. They have not, however, suggested or theorized a better approximation or distribution.

The non-spectral approach operates in the time domain and generally proceeds on a wave-by-wave basis. The advantage of this method is that it includes non-linear





effects not accounted for by spectral analysis. It does, however, have the disadvantage of using more computer time. The time series analysis, as will be shown, has difficulty dealing with the irregularity in height and period of ocean waves, while spectral methods handle this problem quite well.

Although this study uses basically a non-spectral method of analysis to determine wave group boundaries, a spectral analysis was performed to obtain certain wave record parameters to assist in the time series analysis. Thus, the irregularity and the non-linear character of waves should both be taken into account by this combination of the two methods. The spectral analyses also enabled wave group parameters and individual wave parameters to be related back to the spectral analyses themselves.

Time series analysis can be performed with either an analog wave record, which shows all instantaneous sea surface elevations, or with a digitized wave record which shows sea surface elevations at time intervals equal to a fixed sampling rate. Analog wave records must be analyzed by hand, unless they are digitized after the fact, while digitized wave records lend themselves easily to computer analysis.

This study utilized wave records that were digitized and were computer analyzed using the standard zero-upcross



technique. Here, a wave is defined between two successive upcrossings of the wave record with respect to some reference level. The reference level is taken to be the mean water level of the record. As shown in Figure 1, the wave period is defined as the time interval between successive upcrossings while the wave height is defined as the vertical separation between the highest and the lowest point of the wave record between successive upcrossings.

## B. WAVE GROUP DEFINITIONS

As mentioned previously wave groups are an ocean surface phenomenon that is readily apparent to even the most casual observer simply because they represent an orderly succession of wave heights and/or periods against a random background. Even with this being the case, however, there is as yet no common agreement on how wave groups should be defined.

Many definitions of wave groups are in the literature. Most employ readily identifiable wave parameters to define wave-group boundaries. Among the many definitions in use are the largest height of a wave in a group, the height which a sequence of waves exceed, the number of successive waves, the periods of successive waves, and the time that the envelope of a wave record (with the envelope having various definitions) exceeds a certain level. The method



of definition used by Sedivy (1978) used a new approach wherein the energy content of the wave groups was the decisive parameter.

The basic premise used by Sedivy (1978) and in this study is that the energy in a wave record is proportional to the variance of the wave record. The record variance can be calculated from a time-domain analysis or from a frequency-domain analysis. The energy of a simple sinusoidal wave can be shown to be proportional to the square of the height of the wave. Moreover, Michel (1968) states that if the wave field is thought of as the sum of an infinite number of component waves of small amplitude then the energy of the wave field may be considered to be proportional to the sums of the squares of the heights of those component waves. Thus energy, variance, and wave height are related, and if one can evaluate one of these quantities then the other two are obtainable by a proportionality factor.

If the concept is kept in mind that a wave group is essentially a succession of waves which are not random and in general whose heights are larger than the surrounding waves, then wave groups may be thought of as packets whose energy content is greater than that of the surrounding wave field. Over a short period of time, then, the variance of the wave groups will also be larger than the variance of the wave record as a whole.



The wave record is then analyzed by using a window of short duration to look at portions of the wave record. The window slides along the wave record at a rate equal to the digitizing rate, and at each stop the short-term variance is calculated and plotted at the midpoint of the window. The result of this process is illustrated in Figures 2 and 3, where the wave record is shown in the bottom of the figures and the corresponding short-term variance waveform is shown in the top portion of the figures. Those areas under the short-term variance curve that lie above the variance of the wave record (a straight line) denote areas of higher than wave record energy, and hence possible wave groups.

It is noticeable, however, that those areas where the short-term variance waveform crosses the record variance do not often correspond to zero upcrossings of the wave record itself. Realistically and statistically it does not make sense to think of a wave group as being anything else than a series of whole waves. Thus, the short-term variance waveform cannot be used by itself to define wave groups. The definition of a wave group was expanded so that when the boundaries are set on the wave record by the variance waveform analysis, they are then moved away from the center of the wave group to the first zero upcrossing in each direction. This is shown graphically in Figure 4.





Two additional restrictions were imposed on the placement of wave group boundaries. The first restriction is that a wave group must have at least two waves, since one wave was not considered to constitute a group against a random wave field. The second restriction is that wave groups identified as above must be separated by at least one-half the width of the window. Where this condition was not met by two successive wave groups they were treated as one wave group. This restriction was necessary to prevent the possibility of one wave being included in two separate wave groups.

Thompson (1972) and Smith (1974) have shown that the average period of the waves in a group closely approximate the period of maximum energy density in a wave record, or in other words the spectral peak period. Sedivy (1978) used twice this value for the length of the short-term sliding window after experiments on artificially generated wave records showed it to be an optimum value. Experiments on actual wave records for the purpose of this study confirmed this choice of length for the sliding window.

In this study experiments were conducted by varying the window length in fractions and multiples of the spectral peak period. It was found that large energy groups were identified by all windows and that the number of waves in a group was seldom affected by window length. In groups with



small energy, however, the number of groups in a record varied as the window length was varied, and the number of waves in these groups was quite variable. In general as the window length was increased the number of wave groups decreased. The window lengths experimented with ranged from one-half to four times the peak period and over this range the number of wave groups decreased by 50%. This was accompanied by some increase in the number of waves per group. A window length of twice the spectral peak period was therefore used in this study since it identifies all the large energy groups.



### III. WAVE DATA ACQUISITION, SELECTION, AND ANALYSIS

Wave records analyzed in conjunction with this study were provided by the Coastal Engineering Data Network (CEDN) which is sponsored jointly by the California Department of Navigation and Ocean Development (CDNOD) and the U. S. Army Corps of Engineers. Additional funding support is supplied by the University of California Sea Grant Program with Scripps Institution of Oceanography providing overall direction for the actual data collection program.

The wave records received from CEDN were recorded from a Datawell Waverider accelerometer-type buoy anchored in approximately 30 fathoms of water. The buoy was installed by the U. S. Naval Postgraduate School under contract with CDNOD, and is located, as shown in Figure 5, on the open continental shelf approximately four nautical miles south-southwest of the Santa Cruz Point light at  $36^{\circ}53.48'$  North and  $122^{\circ}03.22'$  West. The wave record data were received from CEDN on magnetic tape digitized at a sampling rate of one second. The wave records gave an instantaneous sea level at one second intervals with respect to a mean water level. The mean water level used was the average of all sea surface elevations in the wave record and was assumed not to change significantly over the duration of the record. Each wave record contained 1024 data points (approximately



17-minute duration) with a record generally being recorded synoptically every ten hours. The wave records used in this study were selected from data recorded over a ten month period from June 1978 through March 1979.

CEDN regularly publishes a spectral analysis of the wave records obtained each month in both a graphical and tabular format as shown in Figure 6 and Figure 7, respectively. The wave records used in this study were selected from a review of the published CEDN data on the basis of displaying one main and easily distinguishable spectral peak. Only wave records with unimodal spectra were considered. Multimodal spectra present some complications in the selection of a suitable window length, and it was considered best to examine the properties of wave groups in clearly unimodal spectra first. Thus, individual wave records were initially selected if they displayed more than 30% of the energy in one, and only one, of the CEDN tabular printout spectral period bands. This limit was arbitrarily chosen to ensure that only one prominent energy peak occurred in the spectra, but yet was not so restrictive that only a few wave records would be considered in the study.

CEDN furnished 695 wave records and they were initially searched for single-peaked spectra satisfying the criteria. Approximately half, or 338 of the records, were found to possess the unimodal spectrum desired. These 338 selected





records, the characteristics of which are shown in Figure 10, were analyzed on an IBM 360/67 computer both spectrally and non-spectrally. Examples of the spectral analysis are shown in Figures 8 and 9. This type of analysis was performed in part to obtain better resolution of the spectral peak period than is possible from the two-second period bands provided in the CEDN analysis.

The non-spectral analysis of the wave records utilized the definition of a wave group, as previously defined, to identify wave groups in the wave records and to then compute the various wave record and wave group parameters desired for the statistical portion of the study. Out of 338 wave records 5598 wave groups were so identified and the pertinent statistical parameters generated.

The wave data analyzed in this study may be classified in terms of relative depth as deep water waves or intermediate water waves for the wave record spectral peak periods of interest (four to 18 seconds). Although waves with periods greater than nine seconds have begun to shoal at the location of the wave recorder (30 fathoms), the shoaling coefficient varies only between 0.91 and 1.00. Accordingly all wave records effectively contain deep water waves, and the statistics generated from these records are presumably applicable to the open ocean.



#### IV. WAVE RECORD AND WAVE GROUP PARAMETERS

The wave record and wave group parameters that were either utilized or examined in this study are defined below. Definitions previously used by other authors or definitions that are in common usage are used whenever possible.

##### A. WAVE RECORD PARAMETERS

###### 1. Spectral Peak Period: $T_R$

The spectral peak period of a wave record, as previously mentioned, is that period in the wave record that represents the maximum energy concentration. Sedivy (1978) used an interpolation technique to arrive at  $T_R$  to the nearest second from the two-second bandwidths given in the CEDN spectral printouts. In the present study  $T_R$  was taken as that period corresponding to the frequency of the spectral energy peak obtained from independent spectral analysis of the CEDN wave data. Because of the variation in the resolution of  $T_R$  over the range of wave record periods dealt with,  $T_R$  was taken, for all records, to the nearest whole second.

###### 2. Record Variance: $V_R$

As previously discussed, the variance of a wave record is a measure of the energy in the wave record and is related to the heights of the waves in the record. The



variance can be determined by integration of the power spectrum to obtain the area under the spectral density curve, or it can be calculated directly from the digitized sea surface elevations. The latter method was chosen for two reasons. First, estimates of variance calculated from the time series analysis performed for this study were approximately the same as the variance value from the CEDN spectral printouts and from the independent spectral analysis of CEDN wave data. Second, the sea surface elevation method was easily included in the wave group analysis program at little cost of computer time, and more importantly, is consistent with the group statistics generated by that program.

3. Significant Height:  $H_R$

The significant height of a wave record is defined as the average of the highest one third waves in the wave record, and was calculated from the record variance,  $V_R$ .

4. Wave Steepness:  $G_R$

As defined in Table 1, the steepness parameter for spectrum waves, using  $H_R$  and  $T_R$ , may be defined in an analogous way to the steepness parameter for monochromatic waves. The steepness parameter can be used to generally identify the kind of waves present in the wave record in terms of their relative age, i.e., sea, young swell,



moderate swell, old swell. Thompson and Reynolds (1976) state that since swell steepness diminishes with increasing travel distance from the generating area some measure of the steepness of the wave field,  $G_R$ , can be used for the purpose of estimating relative age and thus the approximate swell distance. The wave steepness parameter is used in this study to determine whether the wave records analyzed are primarily composed of sea or some type of swell, and whether this has some relationship to grouping among the waves. Figure 10 shows the distribution of wave steepness among the 338 wave records analyzed, and shows that with the exception of the old swell band, the wave records are almost evenly distributed over the range of wave age.

#### 5. Record Group Duration: $D_R$

The record group duration is defined as the total amount of time that groups are present over the length of the wave record. It is determined simply by summing the durations of the individual groups, excluding partial groups truncated at the beginning or end of the wave record.

### B. WAVE GROUP PARAMETERS

#### 1. Wave Group Duration: $D$

Wave group duration is the interval of time that a wave group is present, to the nearest second, as determined by the wave group analysis described previously. It is measured directly from the wave record.





2. Number of Waves per Group:  $N_G$

This is the number of whole waves in a wave group of duration  $D$ , and by definition is two or more.

3. Wave Group Period:  $T_G$

The group period is defined as the average period of the individual waves that compose a wave group. One of the distinctive characteristics of wave groups is that the constituent waves tend to be periodic, and  $T_G$  will thus be an approximation of this characteristic. It is calculated by dividing the wave group duration by the number of waves in the group.

4. Average Wave Group Variance:  $V_G$

The average variance of a wave group is a measure of the average energy contained in the wave group and thus is another group parameter of interest. It is calculated by taking an average of the short-term variance values over the length of the wave group from initial to terminal zero upcrossing.

5. Height of the Highest Individual Wave in a Group:  $H_M$

This parameter is defined simply as the height of the single highest wave that occurs in a wave group. It is calculated by the zero upcrossing method (as are all individual wave heights in this study).



### C. WAVE GROUP TO WAVE RECORD PARAMETERS

Three wave group parameters were normalized by taking the ratio of the group parameter to the corresponding record parameter. This allows the group statistics of one wave record to be compared to the group statistics of other wave records.

#### I. Wave Group Variance to Wave Record Variance: $V_G/V_R$

This group/record parameter is the basis for the definition of the wave group, as discussed above; i.e. where  $V_G/V_R$  is greater than one a potential wave group exists. As will be seen in the data, some wave groups have a  $V_G/V_R$  value of less than one. This is explained by the fact that the wave group boundaries are moved outward to include a whole number of waves (Figure 4). This means that for groups with small  $V_G/V_R$  peak values and small numbers of waves that it is possible for more of the short-term variance curve to lie below the record variance curve than above it. As both  $V_G$  and  $V_R$  are measures of energy, their ratio is a dimensionless measure of energy in the group. Thus, the higher the value of  $V_G/V_R$ , the more energy the group contains relative to the wave record. This parameter was investigated as it seems reasonable that the more energy a group contains relative to the wave field surrounding it, the more important is that wave group.



2. Wave Group Period to Spectral Peak Period:  $T_G/T_R$

Thompson (1972), Smith (1974), Thompson and Smith

(1974), and Sedivy (1978) investigated the relationship of  $T_G$  and  $T_R$ . The first three of these papers report that the average group period and the spectral peak period are essentially equal in a wave group. Sedivy (1978) found a tendency for  $T_G/T_R$  to approach a value of one but not to the degree that earlier researchers have found. This parameter was included, in part in the present study, to check this relationship for essentially deep ocean waves; earlier findings were obtained from waves in shallow water recorded by bottom-mounted pressure sensors.

3. Highest Group Wave Height to Significant Wave Height:  $H_M/H_R$

Sedivy (1978) found  $H_M/H_R$  to be strongly dependent

on other group or record parameters, and this parameter was included to help validate those results and look for possible relationships among parameters not considered previously.



## V. RESULTS AND INTERPRETATION

### A. ANALYSIS SCHEDULE

All wave group and wave record parameters examined in this study have been defined previously. Given the large number of possible relationships between them, a systematic method of examination was called for. To facilitate this analysis Table II and Table III were constructed and used as guides. As is shown in each table, a matrix was constructed with all relevant parameters represented (the relationships examined are illustrated in the figures indicated). The possible relationships that are crossed out are either redundant or illogical. Table II represents analyses for all of the wave group data, while Table III illustrates the same analyses for extreme wave groups only.

The data parameters  $T_G/T_R$ ,  $V_G/V_R$ ,  $H_M/H_R$ , and  $G_R$  were divided into bands of values. This alternate view of data parameter distributions was adopted due to the wide range of values possible for each parameter. The band divisions were chosen either on the basis of definitions already in use (i.e., wave record steepness) or to ensure an even distribution of variables in the bands.

All possible relationships in this study were illustrated by means of frequency of occurrence graphs. As will be seen there are two graphs for each relationship, a





percentage distribution curve or histogram, and a cumulative percentage distribution curve. Each type of curve has its own advantages and disadvantages in bringing out the details of each relationship.

The cumulative distribution curves were used to extract quantitative statistical measures for each distribution. Many of the curves approximate a normal distribution. If the distributions were normal then the cumulative 50 percentage level would represent the mean and the cumulative 16 and 84 percentage levels would approximate the first standard deviation about the mean. By analogy, the median of the actual distributions was used to represent the central tendency, and the cumulative 20 and 80 percentage levels were used as an approximation, albeit crude, for the first standard deviation distribution about the mean. In this study, the approximation for the first standard deviation will henceforth be referred to as the standard deviation for all curves examined. The median and the standard deviation (presented below in tabular format) allows rapid and convenient examination of the many distributions that are graphically presented. They also enable one graph to be compared to another graph or to another parameter band of the same graph.



## B. ALL WAVE GROUPS

### 1. Frequency Distribution (graphs)

#### a. $N_G$

The distribution of the number of waves per group for all 338 wave records is shown in the histogram of Figure 11A and in the cumulative percentage distribution of Figure 11B. The modal number of waves per group is three, while the median falls between three and four waves per group. The standard deviation ranges between two and six waves per group. The total range of values of waves per group is from the defined minimum of two to a single group maximum of 28.

The number of waves per group as a function of  $T_G/T_R$  bands is illustrated in Figures 12A and 12B. All  $T_G/T_R$  bands closely resemble the distribution shown previously for all groups (Figure 11A). The large variability of the lowest  $T_G/T_R$  curve is due to the small sample size of the band. The cumulative percentage curve best shows the similarity between the distribution of  $N_G$  for all wave groups and for groups falling within the given  $T_G/T_R$  bands. It also shows that there is no order or gradation among the  $T_G/T_R$  bands. These two sets of curves indicate that there is no dependency of  $N_G$  on  $T_G/T_R$ . Regardless of whether the group period is less than, equal to, or greater than the spectral peak period,  $N_G$  retains the same distribution.



The histograms and cumulative percentage curves for  $N_G$  with respect to relative energy, or  $V_G/V_R$  bands, are shown in Figures 13A and 13B respectively. The most striking feature of the histogram is that as the relative energy in a group increases (i.e., as the  $V_G/V_R$  band value increases), the peak value or the value at which the number of waves per group most frequently occurs shifts to the right toward larger values. The modal number for the lowest  $V_G/V_R$  band is two, while for the highest  $V_G/V_R$  bands it is five or six waves per group. The shape of the curve also changes as  $V_G/V_R$  increases; the curve becomes less peaked and the percentage frequency of occurrence of the peak decreases. The curve widens as it flattens, and a greater total range of values occurs for the higher  $V_G/V_R$  bands. The latter features can also be seen on the cumulative percentage curve. The standard deviation effectively doubles from lowest to highest  $V_G/V_R$  band. The number of waves per group is plainly seen to be related to the relative energy that the group possesses. The greater the relative energy that a group contains, the greater is the average number of waves in the group.

Figures 14A and 14B are the frequency of occurrence graphs for  $N_G$  as a function of  $H_M/H_R$ , the ratio of the highest wave per group to the significant height of the wave record. The curves for  $N_G$  for the  $H_M/H_R$  bands



given show similar characteristics to the curves discussed above for  $N_G$  as a function of  $V_G/V_R$  bands. As the  $H_M/H_R$  bands increase in value, the peak occurrence of  $N_G$  decreases in percentage value and shifts to the right from two to six or seven waves per group. While the curve flattens and spreads out as  $H_M/H_R$  increases, the range of standard deviation approximations also increases in magnitude; the increase being from less than two to two waves per group for the lowest  $H_M/H_R$  band to four to ten waves per group for the highest band. The similarity between  $N_G$  for  $H_M/H_R$  and  $V_G/V_R$  bands is evident as stated (compare Figures 13A and 14A), and logic would dictate that this indication of a relationship between  $H_M/H_R$  and  $V_G/V_R$  is to be expected.  $H_R$  and  $V_R$  are statistical measures that should be proportional, and ratios that contain these two parameters should also be proportional. Thus, the higher the energy content of a group the generally higher should be the waves in the group, including the largest wave  $H_M$ , and the number of waves that the group will possess should be larger.

The distribution of  $N_G$  with respect to wave steepness, and hence relative wave age, shows similarities to the distribution of  $N_G$  for all groups (Figure 11A), but with some important differences. Figure 15A shows that the curves for all four steepness bands are very similar to each other. All are peaked at three waves per group,





although the maximum frequency of occurrence decreases as wave steepness decreases. The most significant difference between steepness bands, however, is best seen in the orderly sequence of the cumulative curves of Figure 15B. The median number of waves per group is seen to increase from three to approximately five in progressing from sea to old swell. The standard deviation varies from a range of two to five waves per group for high steepness waves to two to eight waves per group for low steepness waves. The total range of  $N_G$  values also increases as steepness decreases. Thus, the greater the wave age the greater is the probability of wave groups having a wider range of  $N_G$ , and the probability increases slightly of there being more waves per group. This seems entirely reasonable when one views seas as generally having wider frequency and direction bands compared to swell that have propagated over long distances.

b.  $T_G/T_R$

The  $T_G/T_R$  ratio for all wave groups is shown in Figure 16A and is an approximately normal curve with a peak value at  $T_G/T_R$  of approximately 0.90. This value is a little less than 1.00, which would be the value of  $T_G/T_R$  if the average group period was the same as the spectral peak period as shown by Thompson and Smith (1974). The overall range of  $T_G/T_R$  values is from approximately 0.30 to 3.20. The cumulative percentage curve in Figure 16B shows



that the standard deviation has a small range in value from 0.75 to 1.10 about the median of 0.90. Thus, the probability of getting a value of  $T_G/T_R$  much different from one is small.

The frequency of occurrence of  $T_G/T_R$  as a function of relative energy, expressed by  $V_G/V_R$  bands, is shown in Figure 17A. The distributions for all six  $V_G/V_R$  bands are approximately normal, but with several interesting differences between them. The peak frequency of occurrence of  $T_G/T_R$  increases as the  $V_G/V_R$  bands increase in magnitude. Also, the value of  $T_G/T_R$  for the peak shifts slightly to the right from 0.90 to 1.00 as  $V_G/V_R$  increases. Figure 17B shows that the smaller the magnitude of the  $V_G/V_R$  band the more probable is a slightly wider range of  $T_G/T_R$  values. Thus, the higher the relative energy content of a group, the more probable it is that the  $T_G/T_R$  value will be close to the peak value and also that the peak value will be close to one. Therefore, the tendency among high energy groups is for the average group period to approximately equal the spectral peak period. Since  $T_R$  is fixed for a wave record, this means that for low energy groups the average period will be more variable and will likely be less than  $T_R$ .

The  $T_G/T_R$  distribution by  $H_M/H_R$  bands is illustrated in Figures 18A and 18B. All curves with the



exception of the lowest  $H_M/H_R$  band are approximately normal, and are centered about a median  $T_G/T_R$  value of about 1.00 (Figure 18B). It may also be noted that as  $H_M/H_R$  increases, the probability increases that  $T_G/T_R$  is 1.00 (Figure 18A). The curves for the lower  $H_M/H_R$  bands have quite irregular peaks and they also possess a larger standard deviation. As was the case for  $N_G$ , the frequency of occurrence curves of  $T_G/T_R$  for both  $V_G/V_R$  and  $H_M/H_R$  bands are similar.

The effect of steepness on the  $T_G/T_R$  distribution is shown in Figure 19A. All four curves approximate a normal distribution. The peak percentage values are approximately equal, as is the total range of  $T_G/T_R$  values. It is obvious, however, that  $T_G/T_R$  is a function of steepness, as the distributions show a progressive march to the right for  $T_G/T_R$  as the steepness increases. The band corresponding to low swell ( $G_R < 1/250$ ) peaks between  $T_G/T_R$  values of 0.60 and 0.80, while the band corresponding to seas ( $1/12 < G_R < 1/40$ ) peaks around 1.10. The cumulative percentage curves are shown in Figure 19B. With the exception of the highest steepness band, the three remaining curves are very parallel to each other and the values for the standard deviation are approximately the same. The greater the steepness of a wave record the more probable it is that  $T_G$  will approach  $T_R$  in value for the wave groups contained in the record. However, there is also a little wider range



of possible  $T_G/T_R$  values. This is partly contrary to logic. Old swell of low steepness is expected to have a narrow frequency band. It would seem likely that the range of  $T_G/T_R$  values should thus be small and that  $T_G$  should approximate  $T_R$ . Since this is not the case then how can  $T_R$  be the period of maximum energy density for the record? This question will be looked at again later.

c.  $V_G/V_R$

The frequency of occurrence for the ratio of average wave group variance to record variance for all groups is shown in Figure 20A. The curve is noticeably positively skewed with a very sharp increase to the left of the peak between  $V_G/V_R$  values of 0.80 to 1.00. The peak value occurs at approximately 1.10. The sharp cutoff is simply explained as due to the use of  $V_G/V_R$  equal to one for defining wave groups. If wave groups were defined solely as those parts of the wave record where  $V_G$  was greater than or equal to  $V_R$  there would be no values of  $V_G/V_R$  less than one. As explained earlier, however, other restrictions on the identification of wave group boundaries allow a small number of values less than one. The peak occurring at a value slightly larger than one means that most groups have only just enough relative energy to be included as wave groups. There was a wide range of  $V_G/V_R$  values found, extending from 0.55 to 3.60. The cumulative percentage curve





of Figure 20B reveals that the median value of  $V_G/V_R$  is approximately 1.20 while the standard deviation about that value ranges from 1.00 to 1.50.

Previously described graphs (Figures 13A, 14A, 17A, and 18A) have shown indications of a relationship between  $V_G/V_R$  and  $H_M/H_R$ . This relationship is illustrated in a different way by Figures 21A and 21B. All curves with the exception of the curve for the highest  $H_M/H_R$  band are approximately normal. The irregularity of this curve is the result of a low number of groups in that band. As the  $H_M/H_R$  bands increase in value the percentage occurrence of the peak  $V_G/V_R$  values decreases and the peak itself shifts to the right. The curves also become less peaked, more flattened, and extend over a wider range in the progression toward higher  $H_M/H_R$  values. The cumulative percentage curves illustrate these observations nicely. The standard deviation for  $V_G/V_R$  ranges from 0.90 to 1.05 for the lowest  $H_M/H_R$  band, while for the highest it ranges from 1.75 to 2.45. The principal message obtained from these graphs is that the greater the relative energy in a group, the more likely is the chance that  $H_M$  will exceed  $H_R$ , and by a greater amount. It is also evident that the higher the value of  $H_M/H_R$ , the wider is the range of values possible for  $V_G/V_R$ .

The  $V_G/V_R$  values plotted for steepness bands are shown in Figures 22A and 22B. All four curves are very



nearly statistically identical, and it is clear that wave steepness does not affect  $V_G/V_R$ . In other words,  $V_G/V_R$  will have the same distribution among wave groups regardless of the age or distance traveled of the waves in the wave record.

d.  $H_M/H_R$

The distribution of  $H_M/H_R$  for all wave groups is shown by Figure 23A to approximate a normal curve with a mean close to 1.00. The total range of  $H_M/H_R$  values is approximately between 0.35 and 2.15. The standard deviation obtained from analysis of Figure 23B varies between 0.85 and 1.21. Although a wide range of  $H_M/H_R$  values is possible, it is surprising to note that the probability of occurrence is greatest when  $H_M$  is approximately equal to  $H_R$ . That is, the highest wave in a group will most frequently equal the significant height of the record.

The relationship between  $H_M/H_R$  and steepness is shown by Figures 24A and 24B. The peak occurrence percentage values on the histograms are very nearly the same with the exception of the lowest steepness band which is slightly higher. The value of  $H_M/H_R$  at which the peak occurs shifts to the right as the steepness of the wave record decreases. The cumulative curves also show this shift, however, the standard deviation about the median



remains nearly constant. The range of values in each band also remains almost constant. Figure 21A shows  $H_M/H_R$  and  $V_G/V_R$  to be related, and Figure 24A shows  $H_M/H_R$  and  $G_R$  (steepness) to be related. It is surprising, therefore, to find from Figure 22A that  $V_G/V_R$  and steepness are apparently unrelated.

e.  $D_R$

The length of each wave record was 1024 seconds. The duration or the amount of time that wave groups were present over that time interval is shown in Figures 25A and 25B. The histogram appears quite irregular but the irregularity is magnified in the graphical presentation because of the small duration interval used to generate the histogram. A best-fit curve drawn to this distribution would yield an approximately normal distribution with a mean of about 575 seconds. Thus, on the average, wave groups are present over more than half of the wave record. This is in agreement with the findings of Sedivy (1978). The standard deviation ranges from 485 to 620 seconds.

2. Statistical Measures (tables)

Thus far 15 sets (two each) of frequency of occurrence graphs have been presented, described, and discussed for various wave group parameters. To assist readers in their interpretation, pertinent statistical data on the distributions have been tabulated and are presented in Table IV A-E. The sequence of the data is keyed to the



parameter matrix shown in Table II. The first four columns of numerical data in Table IV A-E contain information from the respective histograms: the parameter value at which the peak occurs, the maximum parameter value, the minimum parameter value, and the total range of parameter values (the maximum value less the minimum value). The next three columns are the 50, 20, and 80 cumulative percentage levels from the cumulative distributions. As described previously, these values represent the median and standard deviation limits, respectively. The next two columns are the 50 minus the 20 cumulative percentage value and the 80 minus the 50 cumulative percentage value. These two columns when compared against each other give a rough idea of the skewness of the distribution. The last column gives the approximate standard deviation range.

### C. EXTREME WAVE GROUPS

The frequency of occurrence graphs examined thus far have utilized as a data base all 5548 wave groups identified in the study. These groups were either examined collectively or in bands according to wave group or wave record characteristics. The engineer may, however, be particularly interested in those groups which are the most likely to damage a structure. These may include groups with a large number of periodic waves that might induce resonance in the





structure, or groups containing a great deal of energy whose waves of large amplitude are capable of exerting excessive fluid forces.

With this in mind, a second series of frequency of occurrence graphs were generated, with the largest wave group in each wave record being used as the data base. Two means of identifying the largest group per record were employed. The largest group was first defined as the group with the largest number of waves, and secondly as the group with the largest  $V_G/V_R$  ratio in each record.

#### 1. Frequency Distribution (graphs)

##### a. Largest $N_G$ per Wave Record

Figure 26A shows the distribution curve for  $N_G$  using only that group in each record having the largest  $N_G$  value. The peak occurrence of the 338 records analyzed was found to be at nine to ten waves per group, with the total range being from five to 28. The curve is approximately normal, but is somewhat positively skewed. The cumulative percentage curve in Figure 26B shows the standard deviation to range between eight and 12 to 13 waves per group.

Figures 27A and 27B present  $T_G/T_R$  distributions for the largest  $N_G$  per wave record. The most striking features of the histogram are the sharpness of the peak and the small range of possible values. The main peak occurs at  $T_G/T_R$  equal to one; however, the median of the distribution



is somewhat less than one. The standard deviation extends from 0.74 to 1.14 and the total range is approximately from 0.50 to 2.10. The distribution shown in Figures 27A and 27B is almost identical to that for all wave groups (Figures 16A and 16B), indicating that  $T_G/T_R$  and  $N_G$  are independent variables. This was also shown in Figures 12A and 12B.

The distribution of  $V_G/V_R$  associated with the largest  $N_G$  per wave record is illustrated in Figures 28A and 28B. The peak occurs at a  $V_G/V_R$  value of about 1.60, although there are secondary maxima on either side of the peak. The distribution is similar to that for all wave groups (Figures 20A and 20B), but is clearly displaced toward larger  $V_G/V_R$  values and is more narrow-banded as indicated by a smaller standard deviation (Tables IV.C and V.A). Thus, those groups in the records with the largest number of waves tend to have higher relative energy values.

The frequency of occurrence of  $H_M/H_R$  with respect to the largest  $N_G$  per wave record is shown in Figures 29A and 29B. The distribution is approximately normal, with the peak being around 1.25 and with overall values ranging from approximately 1.00 to 2.00. As with  $V_G/V_R$ , the distribution of  $H_M/H_R$  among those wave groups with the largest number of waves per wave record is similar to that for all wave groups (Figures 23A and 23B), but is displaced toward larger  $H_M/H_R$  values and is more narrow-banded (Tables IV.D and V.A).



The curves suggest that for groups with large numbers of waves, the highest wave in a group will almost certainly exceed the significant height of the record. In wave records of 17 minute duration,  $H_M$  will tend to be about 30% higher than  $H_R$ .

b. Largest  $V_G/V_R$  per Wave Record

The distribution curves for  $N_G$ , as illustrated in Figure 30A and Figure 30B for the largest group  $V_G/V_R$  value per wave record are generally similar to the curves for the largest group  $N_G$  value per record (Figures 26A and 26B). However, the peak lies in the range of from five to seven waves per group, and the distribution has a slightly larger standard deviation. The long tail to the right in both distributions suggests that those groups with the most waves also have the highest relative energy since it appears that the same groups make up the tail.

Figures 31A and 31B are almost identical in every respect to the corresponding  $T_G/T_R$  graphs generated for the wave groups with largest  $N_G$  per record (Figures 27A and 27B). The description of the latter figures given above therefore describes these figures.

The distribution of  $V_G/V_R$  for the largest group  $V_G/V_R$  per wave record, shown in Figures 32A and 32B, is very similar to that for the largest group  $N_G$  (Figures 28A and 28B), but is displaced substantially toward larger  $V_G/V_R$  values. The histogram shows that the average variance of



the wave group with largest energy in a record may exceed the record variance by a factor of more than three, although a factor of 1.75 to 2.00 is most probable.

Figures 33A and 33B illustrate the histogram and cumulative curves for  $H_M/H_R$  associated with the largest group  $V_G/V_R$  per record. The distribution is very similar to that for wave groups having the largest  $N_G$  per record (Figures 29A and 29B) although it is clearly displaced toward higher  $H_M/H_R$  values (Tables V.A and V.B). The histogram shows that groups having the largest relative energy in 17 minute wave records may have  $H_M/H_R$  values as large as two, although a factor of 1.3 to 1.6 has the greatest probability of occurrence.

## 2. Statistical Measures (tables)

Statistical data on the previous eight sets of frequency of occurrence graphs for extreme wave groups have been tabulated and are presented in Table V.A-B. The organization of the table is the same as previously used (Table IV.A-E), and the same interpretational comments apply.





## VI. SUMMARY

The purpose of this study was to use a digital analysis technique to examine the statistics of selected wave group parameters. Wave groups were defined as packets of waves whose energy density is greater than that of the wave record as a whole. Wave group boundaries were mainly determined by using a sliding window to calculate the short-term variance, which was then compared to the record variance (variance being proportional to energy). The short-term window length was taken to be a function of the spectral peak period. The wave group measures of interest were calculated for each identified wave group. One set of statistics was then generated using all wave groups and another set using selected extreme wave groups only. The distributions and interrelationships of these parameters were represented for analytical purposes by frequency of occurrence graphs.

Wave records were obtained from a Waverider buoy moored in Monterey Bay on the outer continental shelf. The wave records represented deep or intermediate depth waves, and covered a wide range of significant height, spectral peak period, and wave type (represented by wave steepness). Only wave records that displayed a unimodal spectrum were considered for analysis, and 338 of those examined were chosen



for analysis. The analyses were performed on wave records having a duration of 1024 seconds (about 17 minutes). The data base from these records consisted of 5598 wave groups.

The portion of the study dealing with all groups yielded results both interesting and unexpected. It was found that the number of waves per group increases as the relative energy of the wave group increases. It was also shown that the average group period approaches the spectral peak period for groups of high relative energy. The steepness of the wave record has no effect at all on the relative energy of a group, on the other hand, the lower the steepness (hence the greater the distance that the waves have traveled) the larger is the number of waves a group will tend to possess. The height of the highest wave in a group ( $H_M$ ) tends to approximate the significant height of the wave record ( $H_R$ ). The greater the relative energy ( $V_G/V_R$ ) that a group contains, however, the greater is the probability that  $H_M$  will exceed  $H_R$ . Although relationships were shown to exist between relative energy and  $H_M/H_R$ , and between wave steepness ( $G_R$ ) and  $H_M/H_R$ , no relationship could be shown to exist (or the relationship was so subtle as to not be seen) between  $G_R$  and  $V_G/V_R$ .

The extreme wave groups used for statistical analysis were divided into two sets. The first set was composed of that group from each wave record having the largest number



of waves, while the second set contained that group in each record having the largest relative energy content. The main feature of these two analyses was the similarity between the distributions of both data sets for the same parameters. Although the distributions were displaced to the right or left of each other, they were very nearly identical. The average group period of both sets of extreme groups was shown to approximate the spectral peak period ( $T_G/T_R \doteq 1$ ), and the maximum wave height was shown to substantially exceed the significant height of the wave record. It appears that those groups having the largest number of waves were also the same groups that possessed the largest relative energy.



Table 1. Definitions of wave steepness and wave age, applicable to deep water only (Thompson and Reynolds, 1976).

## WAVE STEEPNESS, G

### Monochromatic waves

$$G_M = \frac{H}{L} = \frac{H}{\frac{g}{2\pi} T^2} = \frac{H}{5.12 T^2}$$

H = wave height

L = wave length

T = wave period

### Spectrum waves

$$G_R = \frac{H_R}{\frac{g}{2\pi} T_R^2} = \frac{H_R}{5.12 T_R^2}$$

H<sub>R</sub> = significant wave height

T<sub>R</sub> = spectral peak period

## WAVE AGE (in terms of G<sub>R</sub>)

<u>Wave Age</u>	<u>G<sub>R</sub></u>
Sea	1/12-1/40
Young Swell	1/40-1/100
Moderate Swell	1/100-1/250
Old Swell	1/250+





Table II. Parameter matrix for all wave groups.

	$N_G$	$T_G/T_R$	$V_G/V_R$	$H_M/H_R$	$D_R$
$N_G$					
$T_G/T_R$	Figure 12A Figure 12B				
$V_G/V_R$	Figure 13A Figure 13B	Figure 17A Figure 17B			
$H_M/H_R$	Figure 14A Figure 14B	Figure 18A Figure 18B	Figure 21A Figure 21B		
$G_R$	Figure 15A Figure 15B	Figure 19A Figure 19B	Figure 22A Figure 22B	Figure 24A Figure 24B	
All Groups	Figure 11A Figure 11B	Figure 16A Figure 16B	Figure 20A Figure 20B	Figure 23A Figure 23B	Figure 25A Figure 25B



TABLE III. Parameter matrix for extreme wave groups.

	$N_G$	$T_G/T_R$	$V_G/V_R$	$H_M/H_R$
Largest Group ( $N_G$ )	Figure 26A Figure 26B	Figure 27A Figure 27B	Figure 28A Figure 28B	Figure 29A Figure 29B
Largest Group ( $V_G/V_R$ )	Figure 30A Figure 30B	Figure 31A Figure 31B	Figure 32A Figure 32B	Figure 33A Figure 33B



Table IV.A. Statistical Data for  $N_G$  for all wave groups.

Group Parameter	Figure	Band	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
$T_G/T_R$	11A-11B 12A-12B		3	28	2	26	3.5	2.1	5.9	1.4	2.4	3.3
		0.25-0.49	2	16	2	14	3.4	2.0 <	6.0	-	2.6	-
		0.50-0.74	3	22	2	20	3.7	2.3	6.2	1.4	2.5	3.9
		0.75-0.99	3	28	2	26	3.6	2.1	5.9	1.5	2.3	3.3
		1.00-1.24	3	18	2	16	3.6	2.1	6.0	1.5	2.4	3.9
		1.25-1.49	3	11	2	9	3.4	2.0	5.7	1.4	2.3	3.7
		1.50+	3	10	2	8	3.5	2.1	6.0	1.4	2.5	3.9
$V_G/V_R$	13A-13B	0.75-0.99	2	6	2	4	2.0 <	2.0 <	2.7	-	-	-
		1.00-1.24	3	13	2	11	3.0	2.1	4.5	0.9	1.5	2.4
		1.25-1.49	4	18	2	16	4.5	3.1	6.9	1.4	2.4	3.3
		1.50-1.74	5	20	2	18	5.6	3.7	8.4	1.9	2.6	4.7
		1.75-1.99	5	22	2	20	6.0	4.0	8.4	2.0	2.4	4.4
		2.00+	4	23	3	25	5.4	3.6	7.8	1.8	2.4	-
		0.30-0.79	2	5	2	3	2.0 <	2.0 <	2.0 <	-	-	-
$H_M/H_R$	14A-14B	0.80-0.99	3	12	2	10	2.4	2.0 <	3.4	-	1.0	-
		1.00-1.19	4	14	2	12	3.9	2.7	5.7	1.2	1.8	3.0
		1.20-1.39	5	16	2	14	5.6	3.7	8.2	1.9	2.6	4.5
		1.40-1.59	7	20	3	17	6.7	4.5	9.2	2.2	2.5	4.7
		1.60+	6	28	3	25	6.9	4.6	9.9	2.3	3.0	5.3
		1/12-1/40	3	15	2	13	3.2	2.0 <	5.2	-	2.0	-
		1/40-1/100	3	18	2	16	3.4	2.1	5.9	1.3	2.5	3.8
$G_R$	15A-15B	1/100-1/250	3	22	2	20	3.8	2.3	6.5	1.5	2.7	4.2
		1/250+	3	28	2	26	4.2	2.4	7.8	1.8	3.6	5.4



Table IV.B. Statistical Data for  $T_G/T_R$  for all wave groups.

Group Parameter	Figure	Band	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	50-50%	Std.Dev. Range
All Groups	16A-16B 17A-17B		0.90	3.20	0.30	2.90	0.90	0.72	1.10	0.19	0.20	0.35
		0.75-0.99	0.90	3.10	0.30	2.80	0.90	0.68	1.12	0.22	0.22	0.41
		1.00-1.24	0.90	2.80	0.40	2.40	0.88	0.70	1.06	0.18	0.18	0.36
		1.25-1.49	0.90	3.20	0.50	2.70	0.94	0.74	1.03	0.20	0.14	0.34
		1.50-1.74	0.90	2.40	0.40	2.00	0.94	0.76	1.03	0.19	0.14	0.32
$V_G/V_R$		1.75-1.99	1.00	2.60	0.60	2.00	0.97	0.82	1.05	0.05	0.11	0.16
		2.00+	1.00	2.00	0.60	1.40	1.00	0.82	1.03	0.18	0.03	0.26
		0.30-0.79	1.10	3.10	0.30	2.80	0.92	0.64	1.16	0.23	0.24	0.32
		0.80-0.99	0.90	3.20	0.40	2.80	0.94	0.73	1.13	0.21	0.19	0.40
		1.00-1.19	0.90	2.40	0.40	2.00	0.90	0.71	1.08	0.19	0.18	0.37
$H_M/H_R$	18A-18B	1.20-1.39	0.90	2.00	0.50	1.50	0.92	0.74	1.03	0.19	0.16	0.34
		1.40-1.59	1.00	1.90	0.40	1.50	0.91	0.76	1.05	0.15	0.14	0.29
		1.60+	1.00	1.80	0.40	1.40	0.94	0.77	1.05	0.17	0.11	0.23
		1/12-1/40	1.10	3.20	0.40	2.80	1.08	0.90	1.41	0.18	0.33	0.51
		1/40-1/100	0.90	2.40	0.30	2.10	0.92	0.74	1.03	0.18	0.16	0.34
$G_R$	19A-19B	1/100-1/250	0.90	1.70	0.35	1.35	0.80	0.65	0.95	0.15	0.15	0.30
		1/250+	0.60	1.15	0.30	0.85	0.67	0.52	0.83	0.15	0.16	0.31





Table IV.C. Statistical Data for  $V_G/V_R$  for all wave groups.

Group Parameter	Figure	Band	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
All Groups	20A-20B 21A-21B	0.30-0.79 0.80-0.99 1.00-1.19 1.20-1.39 1.40-1.59 1.60+	1.10	3.55	0.50	3.05	1.20	1.00	1.52	0.20	0.32	0.52
			1.00	1.50	0.50	1.00	0.93	0.85	1.03	0.08	0.10	0.13
			1.10	1.70	0.70	1.00	1.04	0.94	1.15	0.10	0.11	0.21
			1.30	2.10	0.90	1.20	1.24	1.10	1.40	0.14	0.16	0.30
			1.60	2.40	1.10	1.30	1.52	1.33	1.70	0.19	0.18	0.37
$H_M/H_R$		1.20-1.39 1.40-1.59 1.60+	1.60	2.80	1.30	1.50	1.74	1.56	2.00	0.18	0.26	0.44
			2.40	3.55	1.40	2.15	2.12	1.74	2.42	0.38	0.30	0.63
			1.10	3.30	0.50	2.80	1.20	1.00	1.50	0.20	0.30	0.50
			1.10	3.55	0.65	2.90	1.20	1.02	1.50	0.18	0.30	0.45
			1.10	3.25	0.75	2.50	1.22	1.02	1.53	0.20	0.31	0.51
$G_R$	22A-22B	1/12-1/40 1/40-1/100 1/100-1/250 1/250+	1.10	2.85	0.85	2.00	1.16	1.02	1.44	0.14	0.28	0.42

Table IV.D. Statistical Data for  $H_M/H_R$  for all wave groups.

All Groups	23A-23B 24A-24B	1/12-1/40 1/40-1/100 1/100-1/250 1/250+	1.05 0.95 0.95 1.05 1.05	2.15 1.75 1.95 2.15 2.05	0.35 0.35 0.35 0.45 0.55	1.80 1.40 1.60 1.70 1.50	1.01 0.95 1.01 1.05 1.05	0.83 0.79 0.84 0.87 0.90	1.20 1.14 1.20 1.24 1.27	0.18 0.16 0.17 0.18 0.15	0.19 0.19 0.19 0.19 0.22	0.37 0.35 0.36 0.37 0.37
------------	--------------------	--	--------------------------------------	--------------------------------------	--------------------------------------	--------------------------------------	--------------------------------------	--------------------------------------	--------------------------------------	--------------------------------------	--------------------------------------	--------------------------------------

Table IV.E. Statistical Data for  $D_R$  for all wave groups.

All Groups	25A-25B		575.0	760.0	290.0	470.0	555.0	485.0	615.0	70.0	60.0	130.0
------------	---------	--	-------	-------	-------	-------	-------	-------	-------	------	------	-------



Table V.A. Statistical data for extreme wave groups: largest  $N_G$  per wave record.

Group Parameter	Figure	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
$N_G$	26A-26B	9	28	5	23	9.8	8.1	12.4	1.7	2.6	4.3
$T_G/T_R$	27A-27B	1.00	2.10	0.50	1.50	0.88	0.74	1.14	0.14	0.26	0.40
$V_G/V_R$	28A-28B	1.60	2.60	1.20	1.40	1.57	1.36	1.80	0.21	0.23	0.44
$H_G/H_R$	29A-29B	1.25	2.15	0.95	1.20	1.23	1.14	1.42	0.14	0.14	0.28

Table V.B. Statistical data for extreme wave groups: largest  $V_G/V_R$  per wave record.

Group Parameter	Figure	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
$N_G$	30A-30B	7	28	3	25	6.3	4.2	9.0	2.1	2.7	4.8
$T_G/T_R$	31A-31B	1.00	2.60	0.50	2.10	0.92	0.76	1.07	0.16	0.15	0.31
$V_G/V_R$	32A-32B	1.80	3.55	1.40	2.15	1.90	1.73	2.13	0.17	0.28	0.45
$H_G/H_R$	33A-33B	1.55	1.95	1.05	0.90	1.42	1.25	1.56	0.17	0.14	0.31



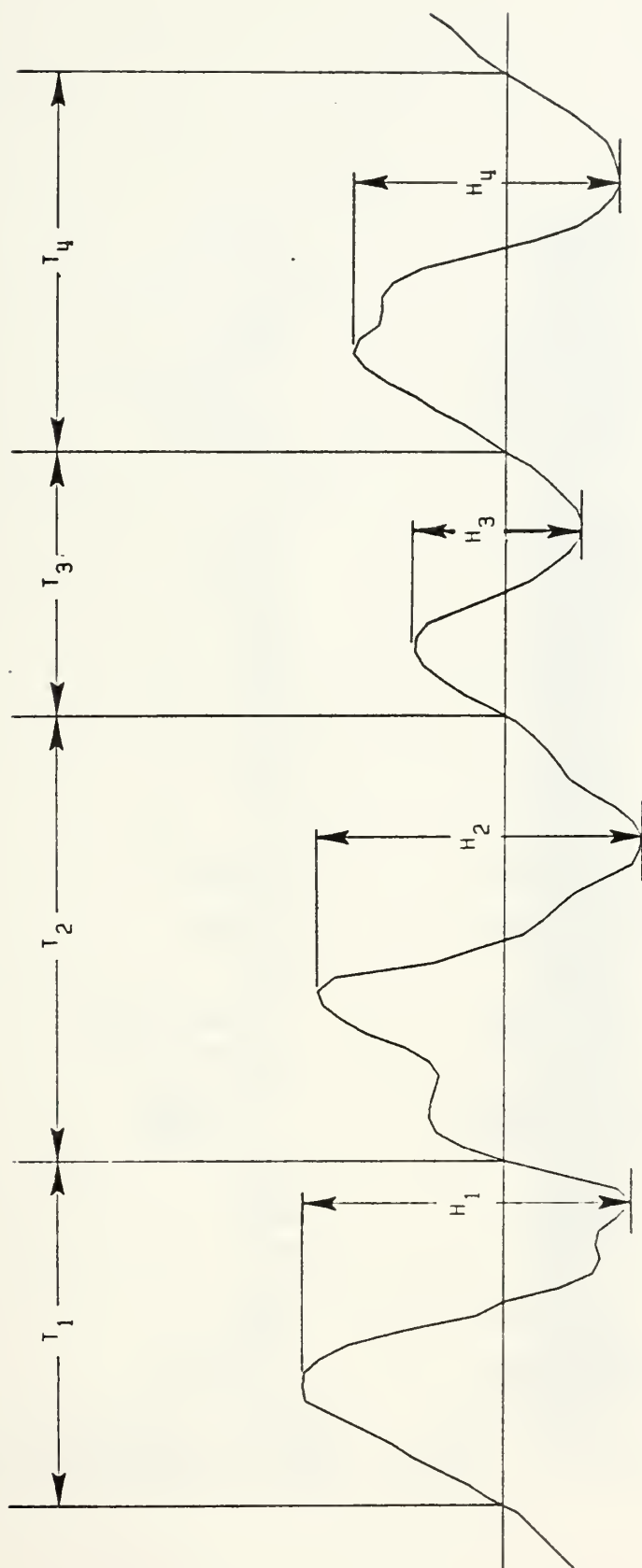


Figure 1. Illustration of zero upcrossing method definitions for wave height and period.



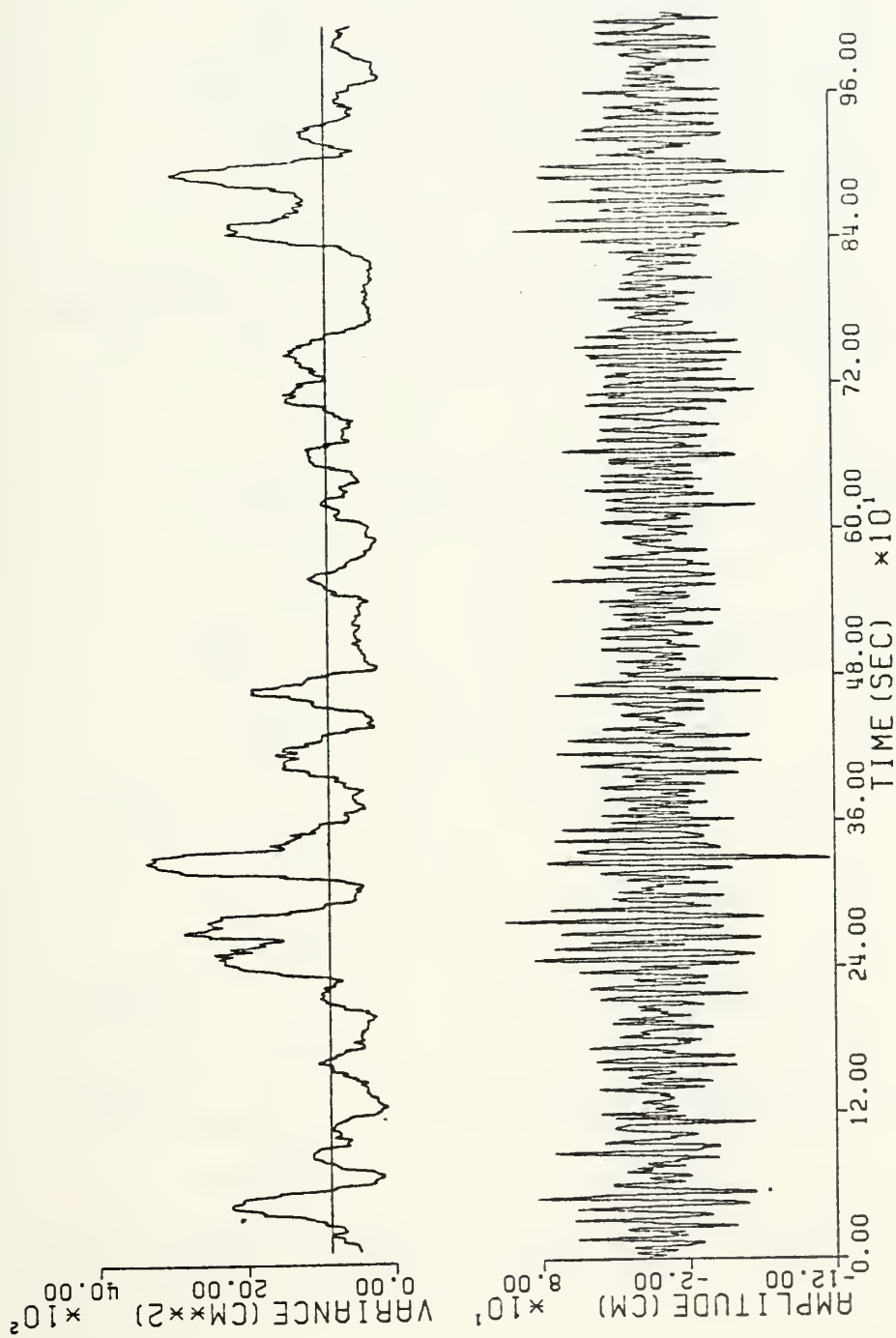


Figure 2. Wave record and group waveform analysis for wave record number 143.





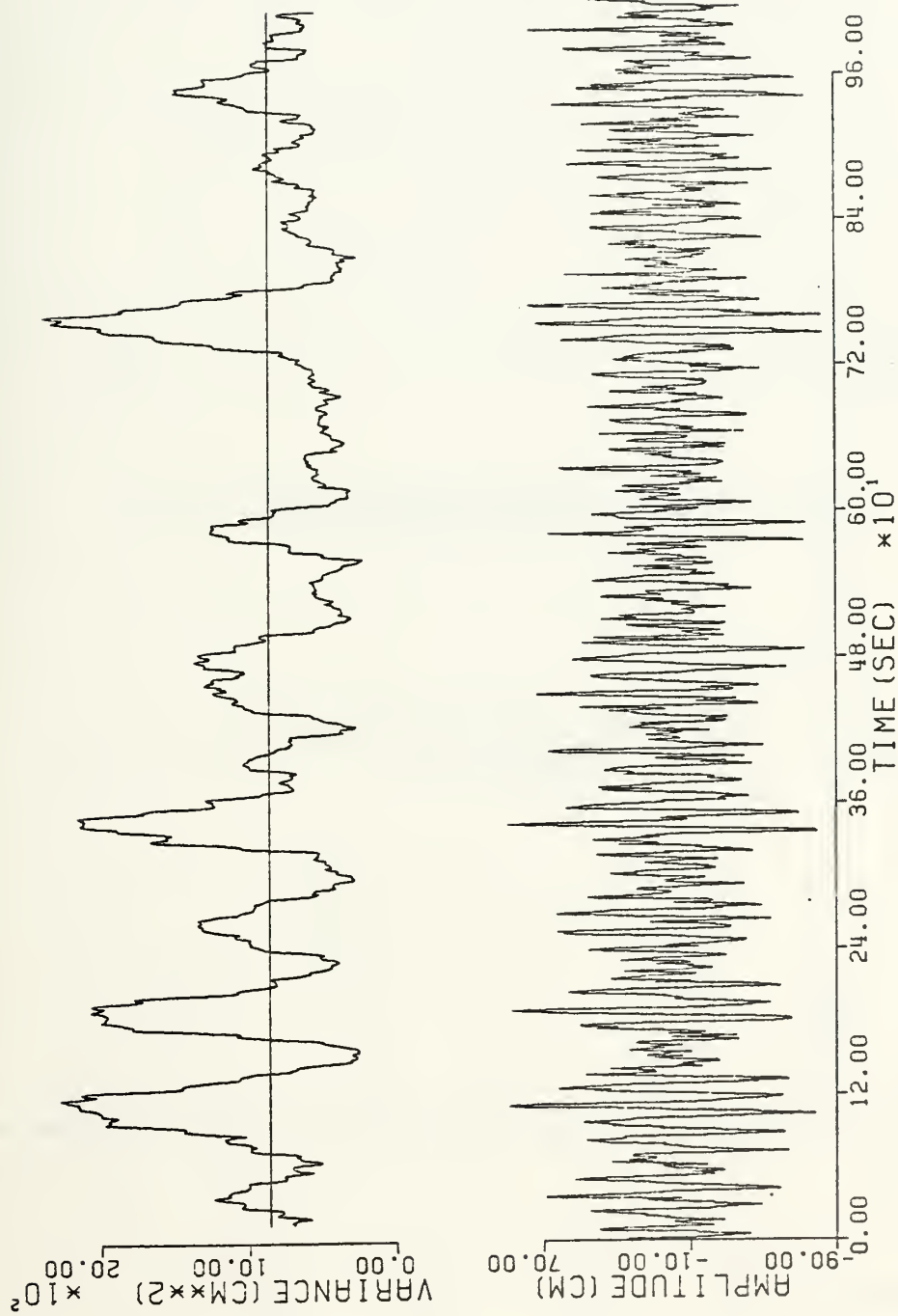


Figure 3. Wave record and group waveform analysis for wave record number 285.



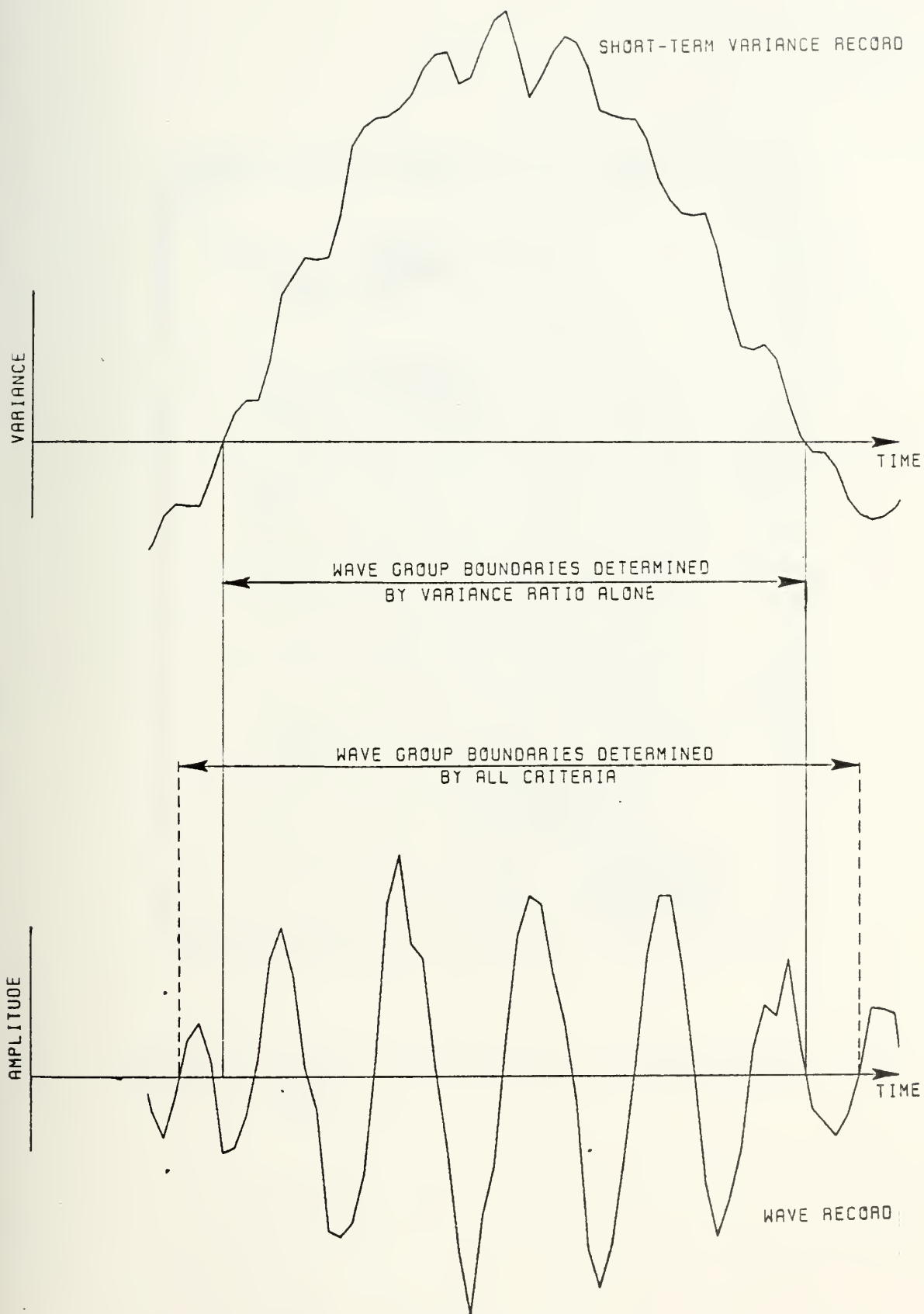


Figure 4. Illustration of wave group boundary definition.



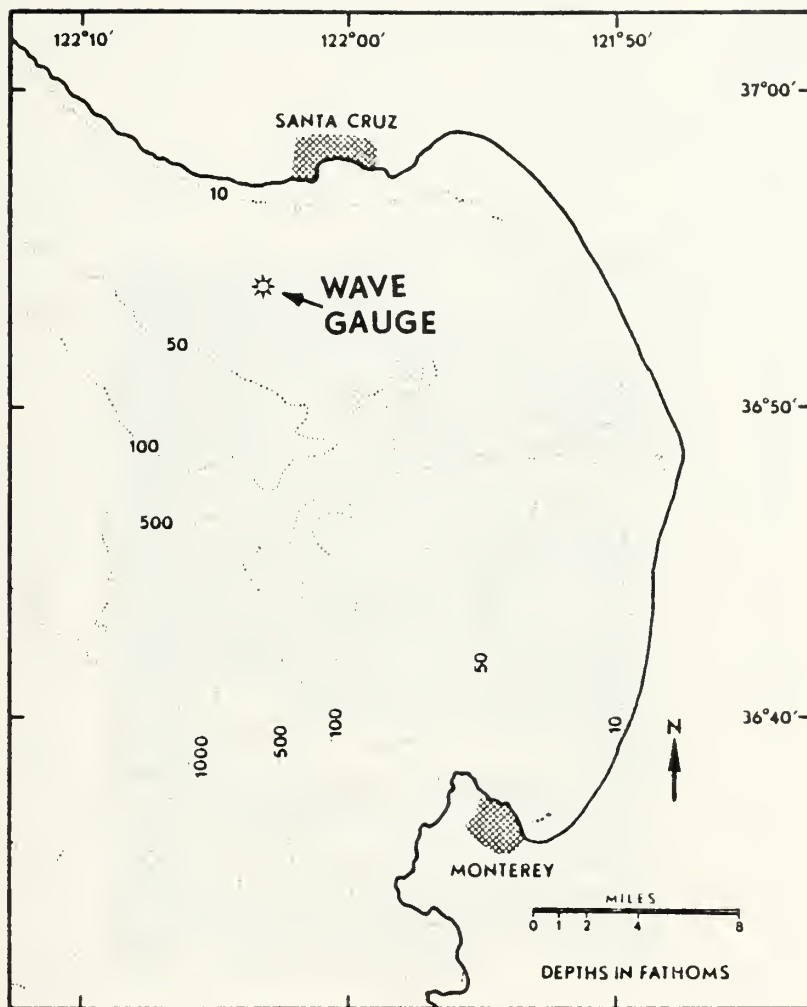


Figure 5. Location of Monterey Bay Waverider Buoy.



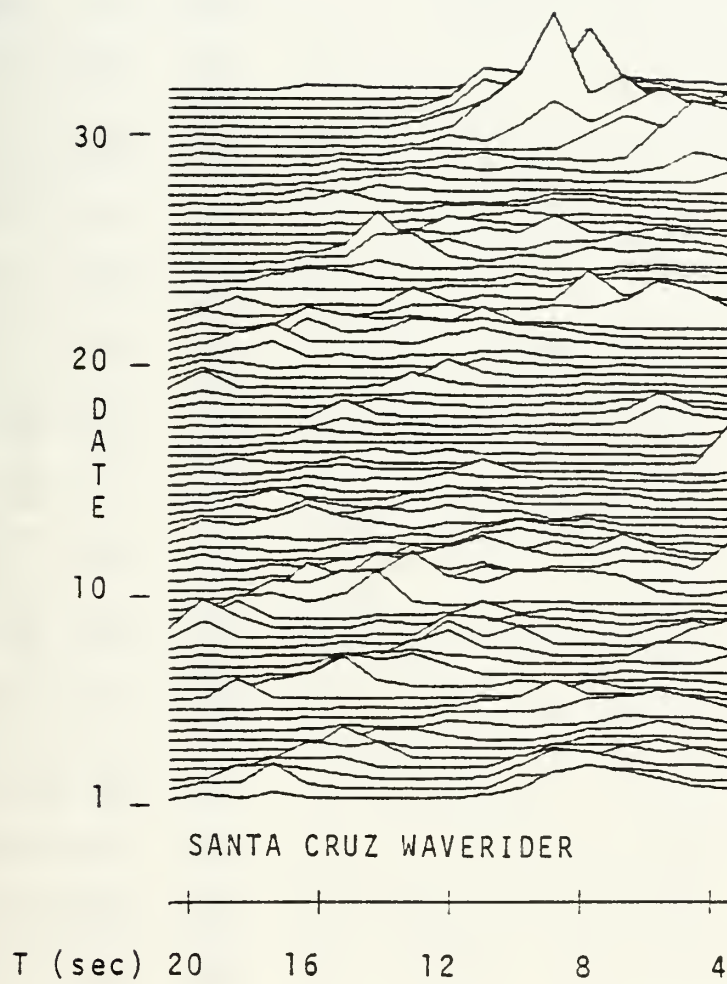


Figure 6. Example of CEDN graphical spectra presentation.





SANTA CRUZ PIER WAVERIDER  
OCT. 1978

PERCENT ENERGY IN BAND  
(TOTAL ENERGY INCLUDES RANGE 2048-4 SECS)

LOCAL DAY/TIME	SIG. HT (CM)	TOT. EN (CM. SQ)	BAND PERIOD LIMITS (IN SECS)									
			22+	22-18	18-16	16-14	14-12	12-10	10-8	8-6	6-4	
1 0640	130.9	1070.2	0.2	3.7	4.7	0.8	1.4	10.8	30.0	20.0	12.5	
1 1640	123.3	950.1	0.2	9.3	21.3	0.4	0.9	11.7	19.2	25.6	11.5	
2 0240	130.7	1201.5	0.4	0.6	13.6	2.3	1.5	10.7	30.0	17.6	14.4	
2 1240	130.3	1195.9	0.4	2.3	13.8	7.9	2.1	9.7	20.5	24.0	19.3	
2 2240	126.9	1006.9	0.2	0.6	14.2	20.0	2.5	2.8	20.4	24.0	14.7	
3 0042	113.6	806.6	0.2	1.3	5.0	25.9	4.3	2.8	18.3	25.1	17.1	
3 1040	110.0	767.0	0.4	1.9	2.7	12.9	14.6	2.5	16.1	30.6	18.3	
4 0440	110.0	767.0	0.9	0.9	4.5	10.2	13.7	9.7	15.6	26.1	18.4	
4 1736	133.0	1104.9	0.1	1.2	2.9	2.0	7.6	11.5	21.3	33.2	19.3	
5 0333	123.1	947.3	0.1	0.7	2.7	5.2	5.4	19.3	31.1	20.1	16.4	
5 1347	112.0	784.5	0.2	21.7	2.2	3.6	5.3	7.4	25.9	21.2	12.4	
6 0002	122.0	930.5	0.1	1.0	20.2	29.6	3.6	0.7	11.7	15.4	0.0	
6 1207	112.9	796.1	0.2	1.4	7.9	33.6	29.9	4.9	5.0	9.6	6.7	
6 2207	114.3	816.7	1.1	0.5	5.5	26.4	24.5	11.5	6.5	9.7	14.3	
7 0007	110.7	801.3	1.0	1.2	4.0	6.9	35.0	14.6	2.0	15.0	16.2	
7 1007	126.9	1006.9	0.6	1.5	3.2	7.4	23.1	21.9	5.0	5.5	31.0	
8 0407	125.0	977.2	0.2	11.4	1.5	6.2	18.3	21.2	12.6	10.5	10.1	
8 1407	122.0	943.0	0.3	20.9	0.9	0.1	9.5	17.0	11.3	9.7	14.3	
9 0007	111.3	773.0	0.3	20.6	1.3	6.3	11.5	22.1	16.2	9.1	12.6	
9 1157	129.9	1054.3	0.4	9.6	15.3	36.0	6.0	10.2	7.9	6.9	6.9	
9 2150	105.6	2153.1	0.2	1.6	11.7	18.6	22.1	14.3	18.2	9.2	4.0	
10 0756	151.0	1425.3	0.3	1.5	14.7	15.3	19.0	20.2	12.7	0.0	0.3	
10 1640	152.3	1450.0	0.3	1.3	3.0	15.2	14.2	11.7	13.3	17.5	22.6	
11 0240	161.2	1624.6	0.7	0.9	4.6	13.3	17.9	23.3	13.3	16.0	9.2	
11 1240	135.5	1146.7	1.3	3.9	3.1	5.7	15.3	17.3	19.4	24.1	10.0	
11 2240	130.6	1066.5	1.0	4.0	2.0	12.0	9.5	27.4	18.4	14.7	9.5	
12 0040	123.6	954.0	1.3	6.3	5.0	7.0	10.4	20.0	22.7	10.0	0.6	
12 1040	139.2	1210.0	0.5	11.1	24.0	12.6	0.3	13.6	14.0	0.2	6.0	
13 0440	122.3	935.3	0.5	0.0	7.1	19.7	21.5	10.1	9.5	0.3	7.4	
13 1514	122.3	935.2	0.4	9.2	12.1	0.1	23.9	19.9	12.7	0.2	5.6	
14 0113	121.5	922.6	0.6	6.3	15.4	7.2	24.1	15.6	13.3	10.0	7.4	
14 1113	97.5	594.6	0.4	5.0	19.0	7.5	10.7	16.6	20.9	9.6	10.3	

Figure 7. Example of CEDN tabular spectra presentation.



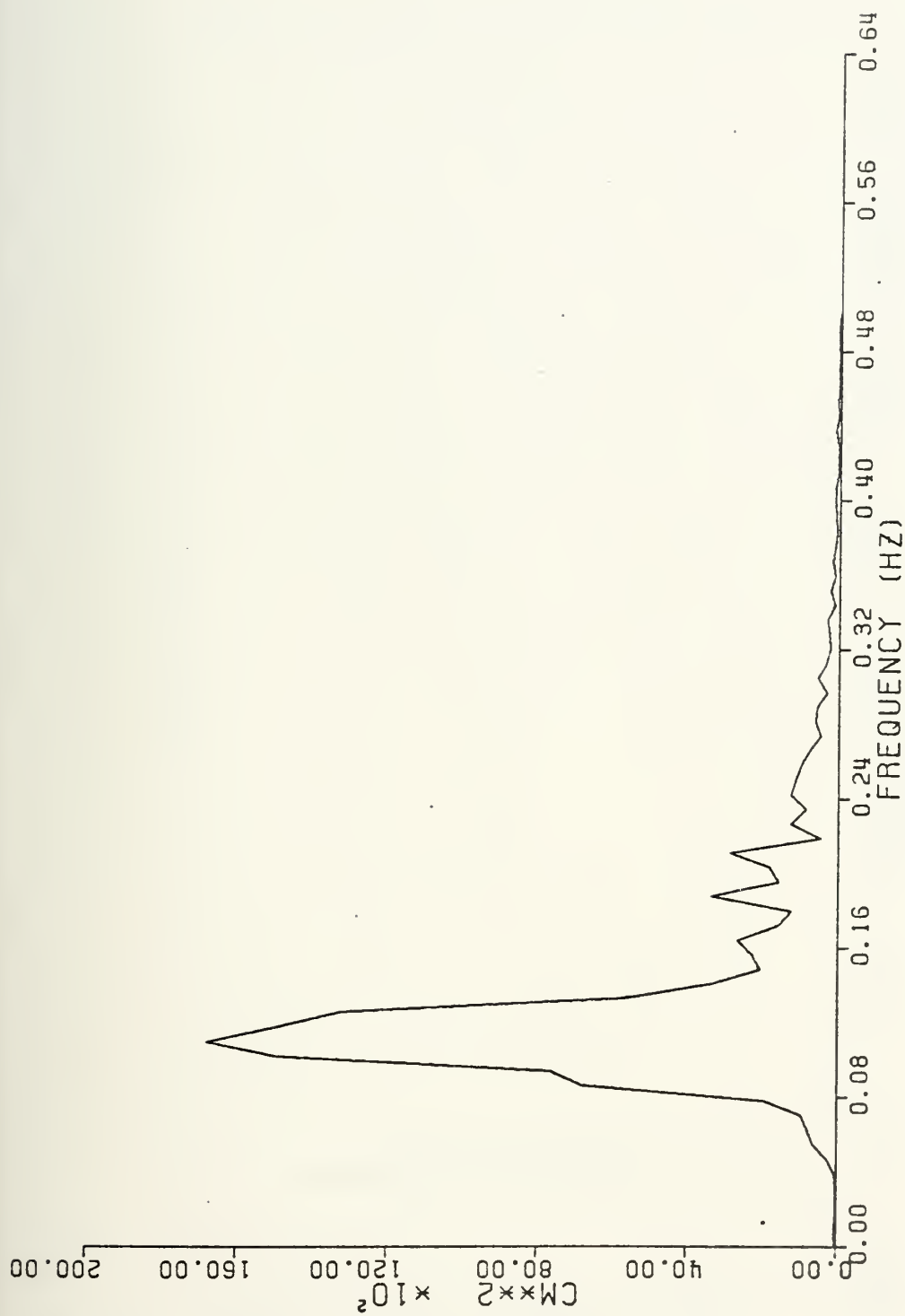


Figure 8. Spectral analysis of wave record number 143.



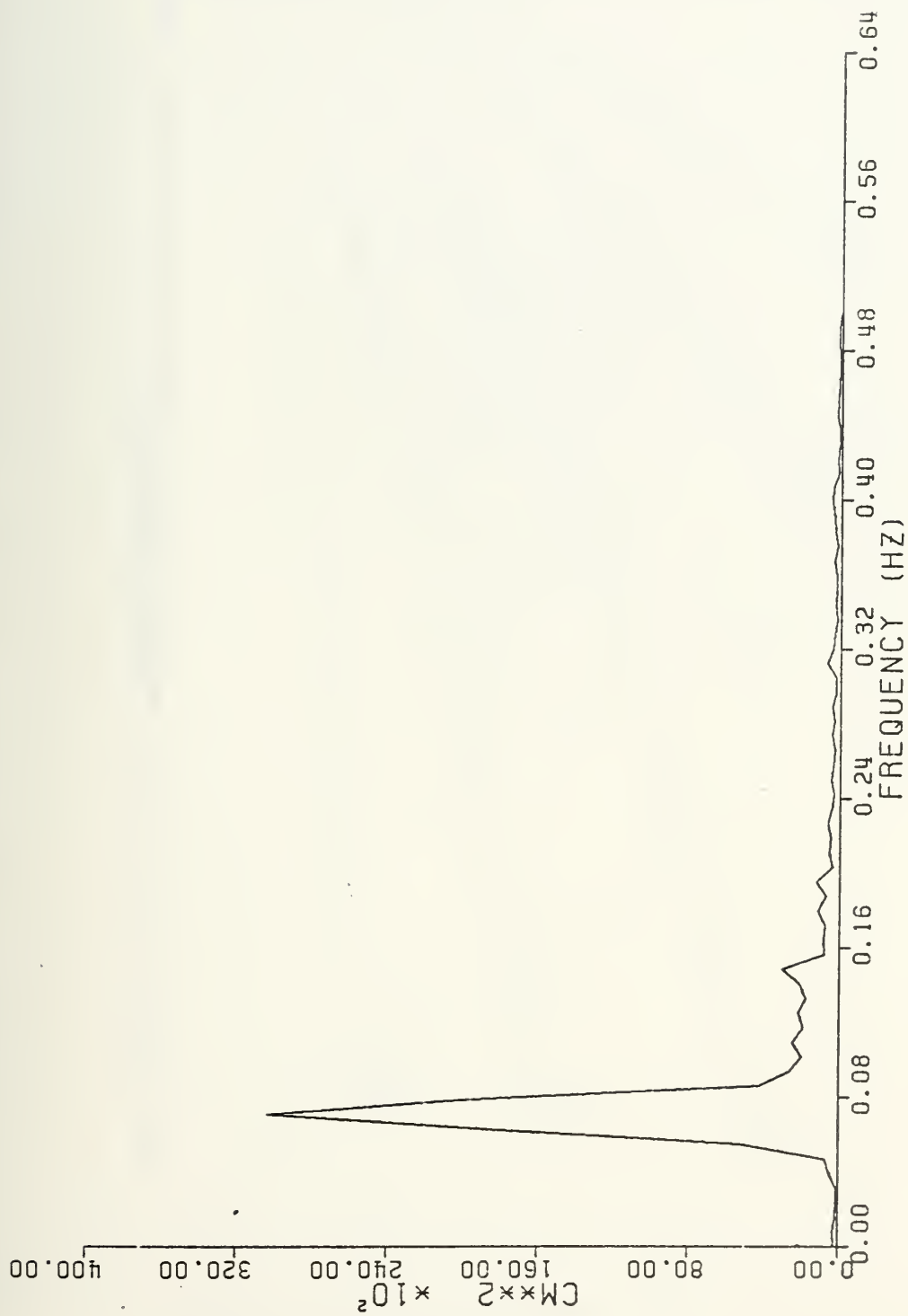


Figure 9. Spectral analysis of wave record number 285.





Figure 10. Height, period, and steepness distribution of wave records sampled.





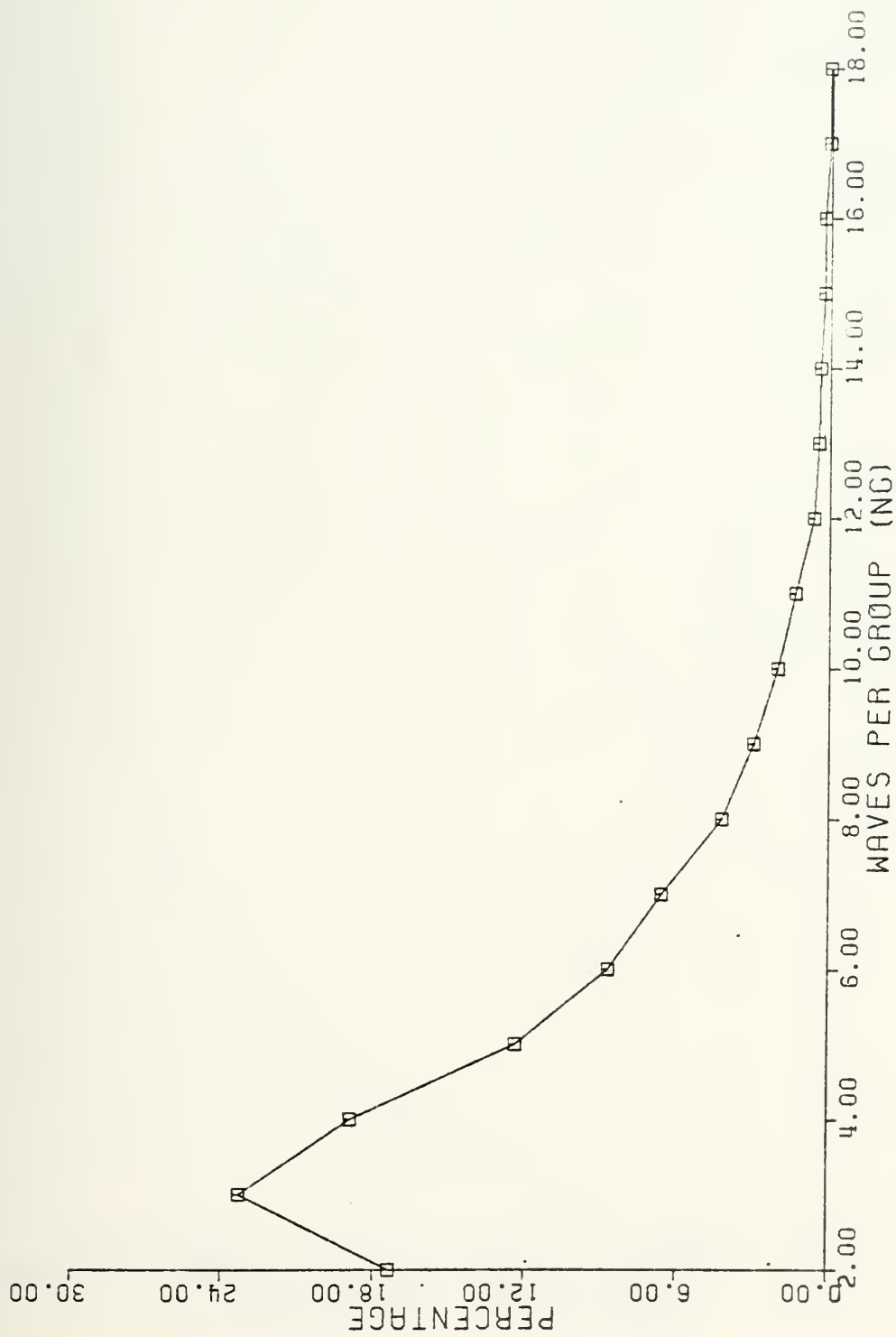


Figure 11A. Histogram of  $N_G$  for all wave groups.



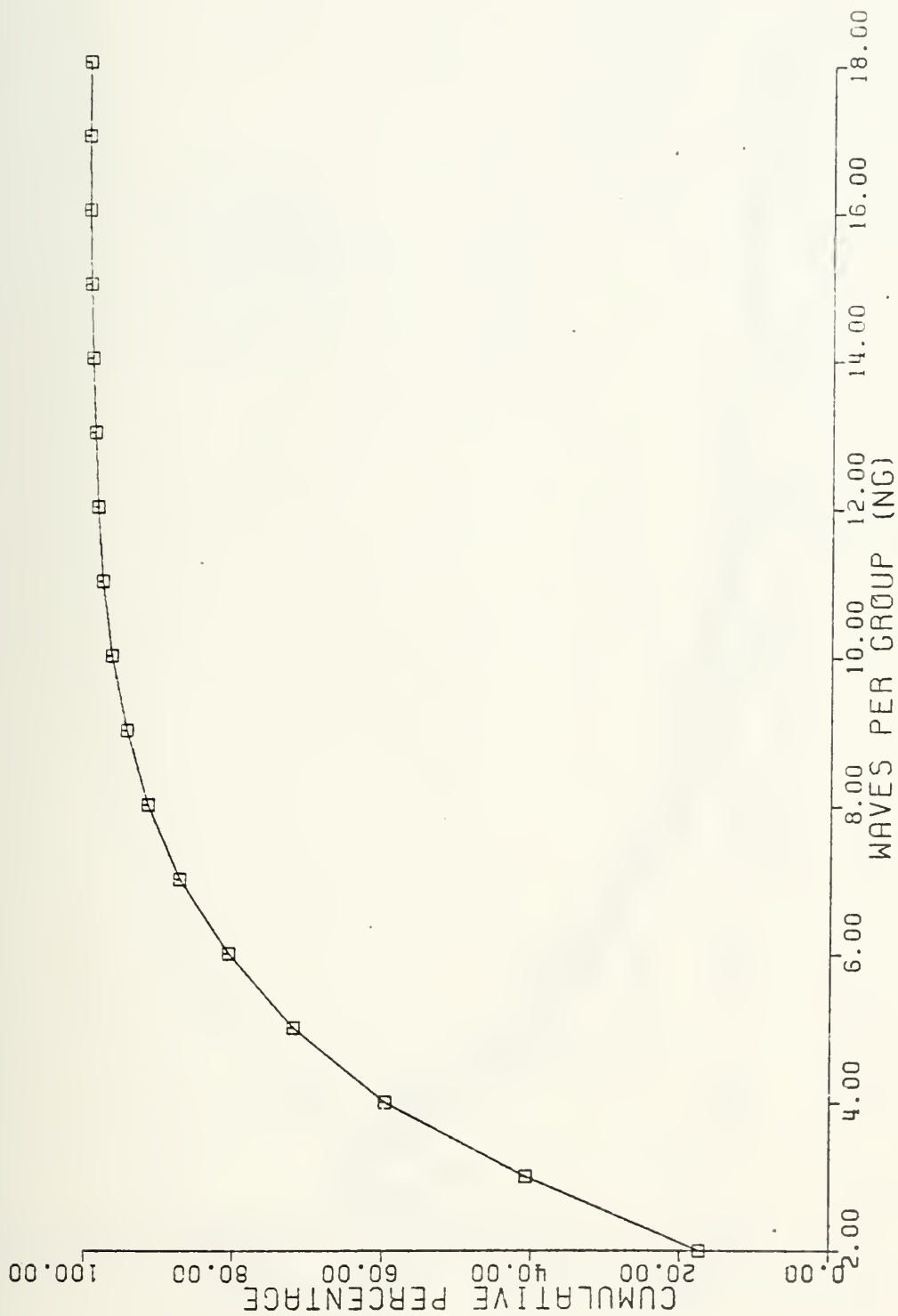


Figure 11B. Cumulative percentage distribution of  $N_G$  for all wave groups.



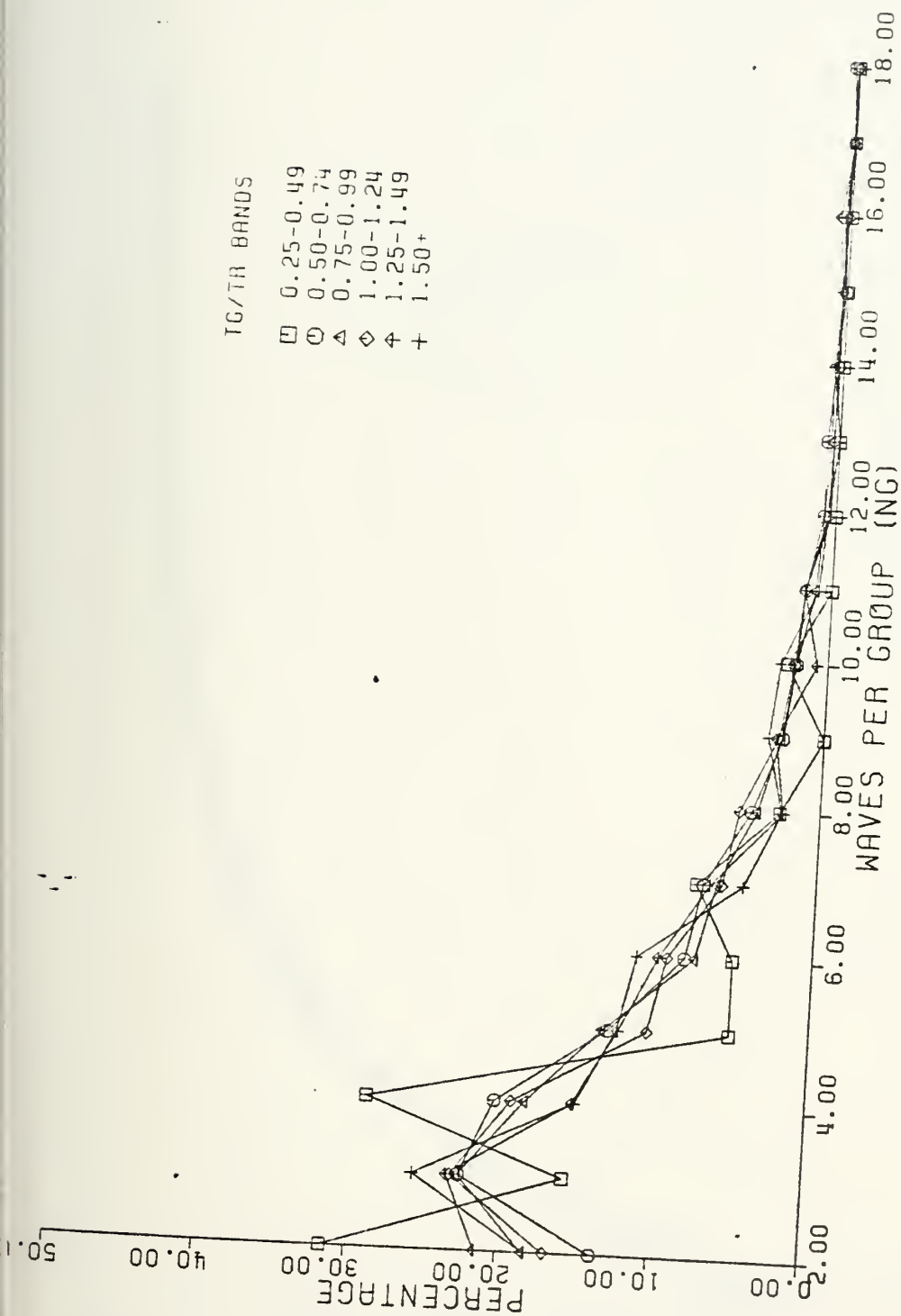


Figure 12A. Histogram of  $N_G$  for  $T_G/T_R$  wave group bands.



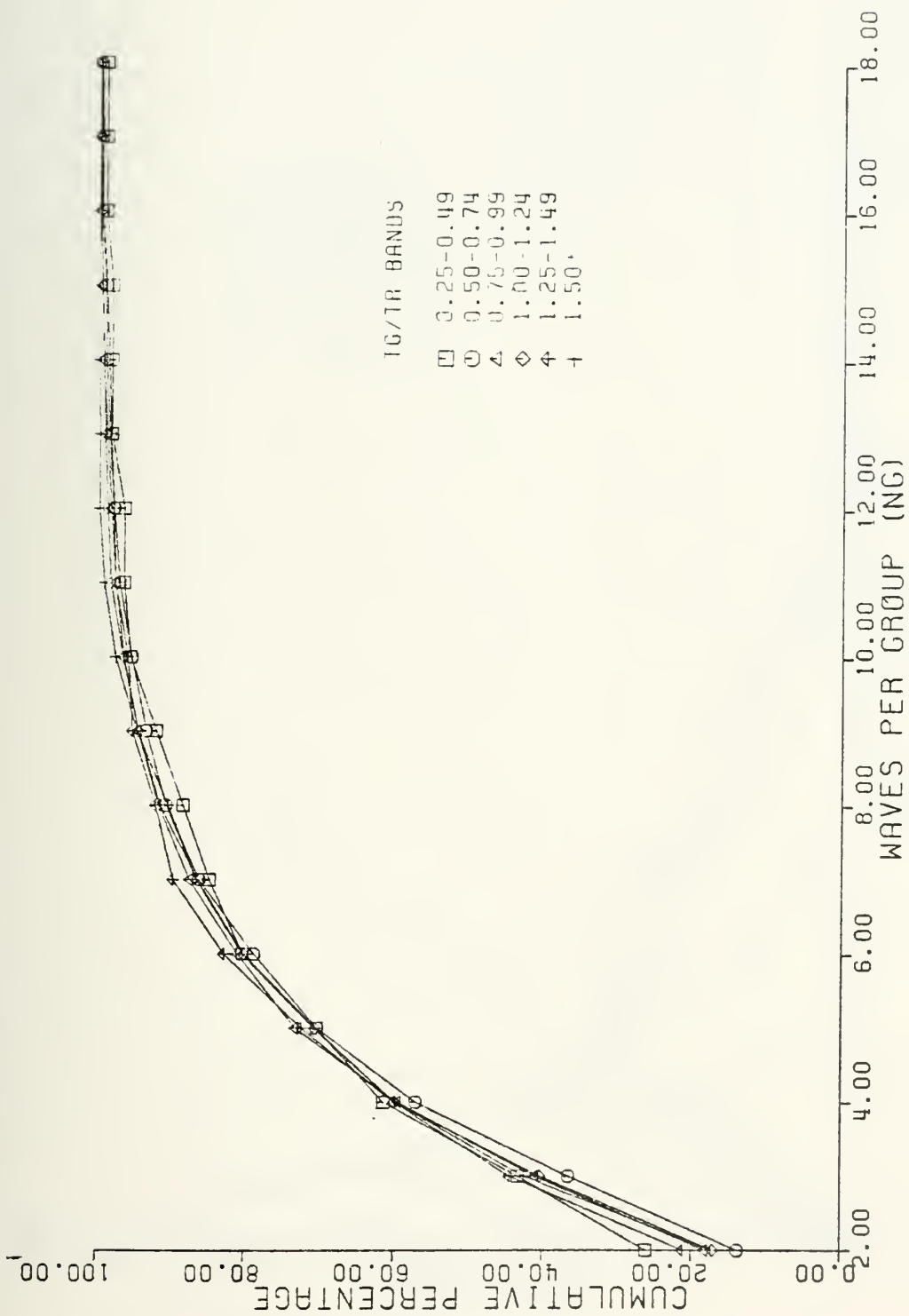


Figure 12B. Cumulative percentage distribution of  $N_G$  to  $T_G/T_R$  wave group bands.





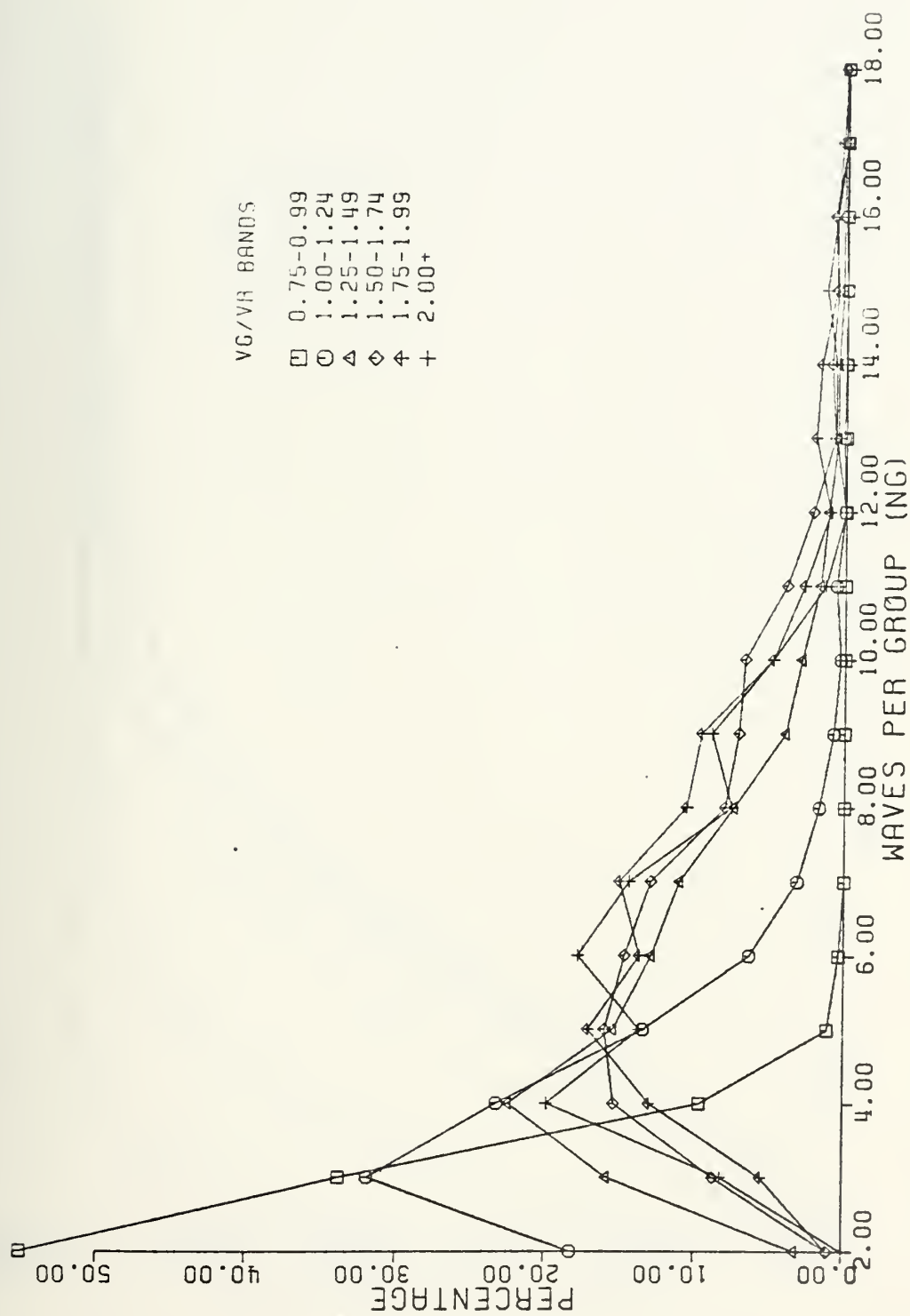


Figure 13A. Histogram of  $N_G$  for  $V_G/V_R$  wave group bands.



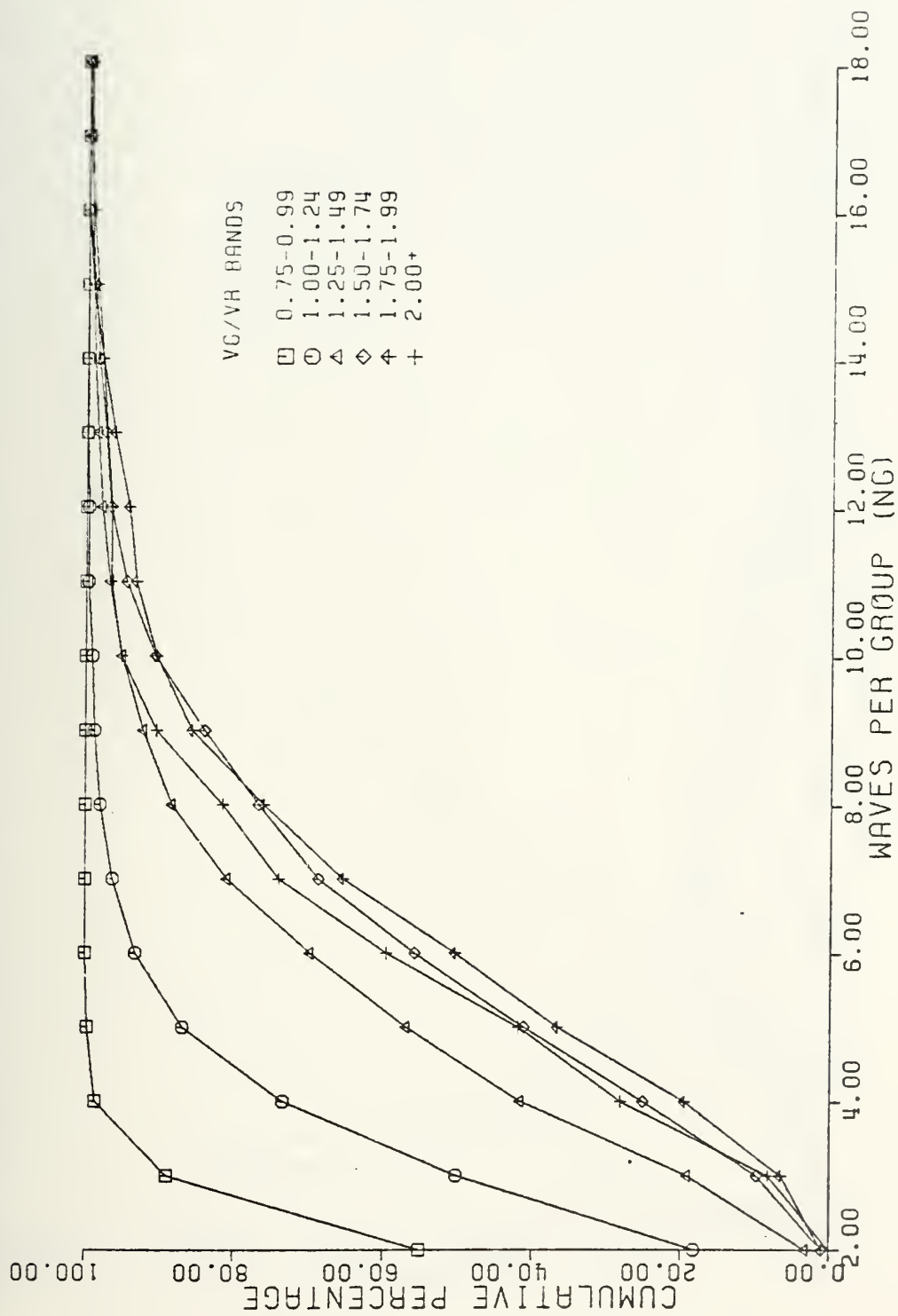


Figure 13B. Cumulative percentage distribution of  $N_G$  for  $V_G/V_R$  wave group bands.



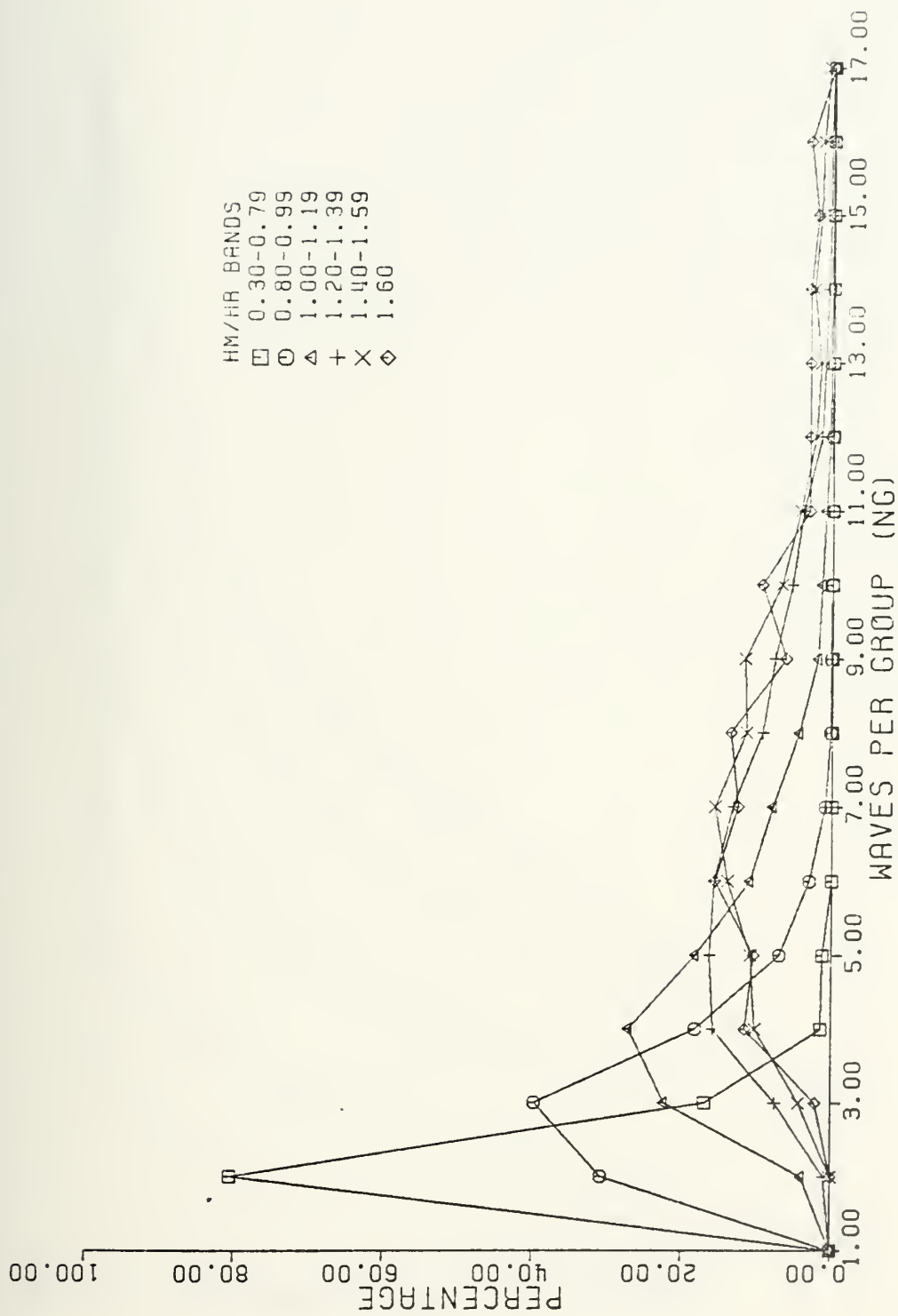


Figure 14A. Histogram of  $N_G$  for  $H_M/H_R$  wave group bands.



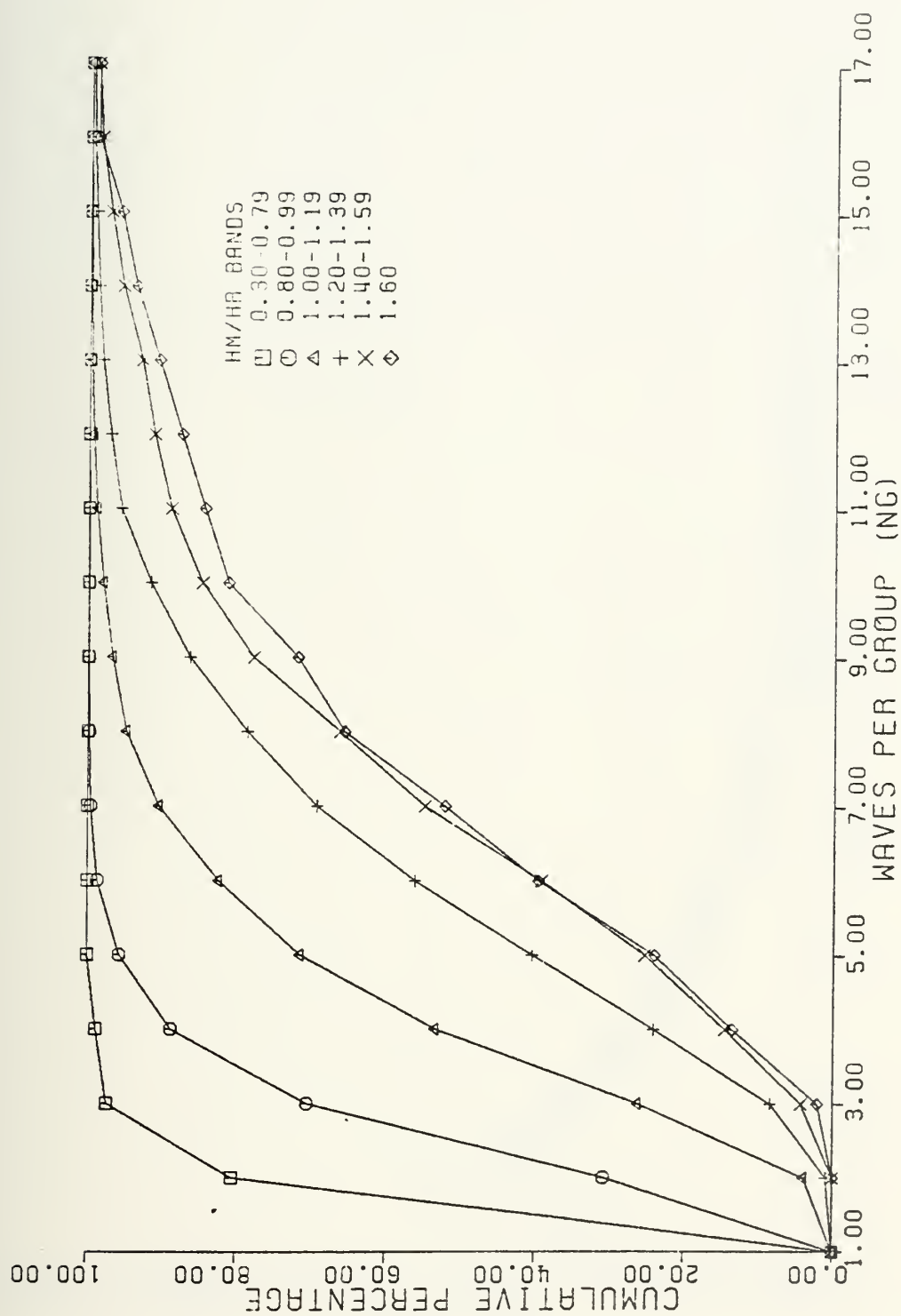


Figure 14B. Cumulative percentage distribution of  $N_G$  for  $H_M/H_R$  wave group bands.





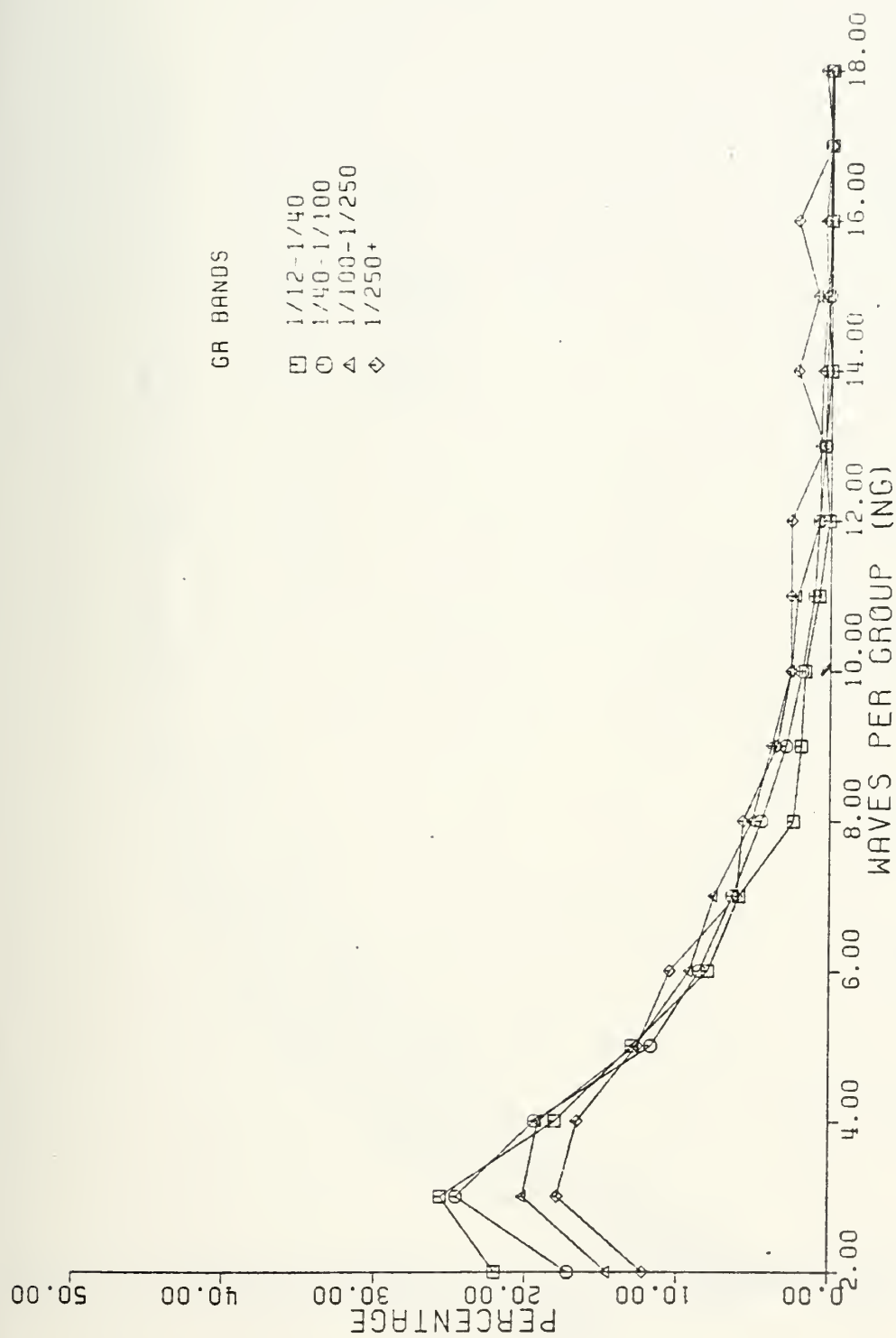


Figure 15A. Histogram of  $N_G$  for  $G_R$  wave group bands.



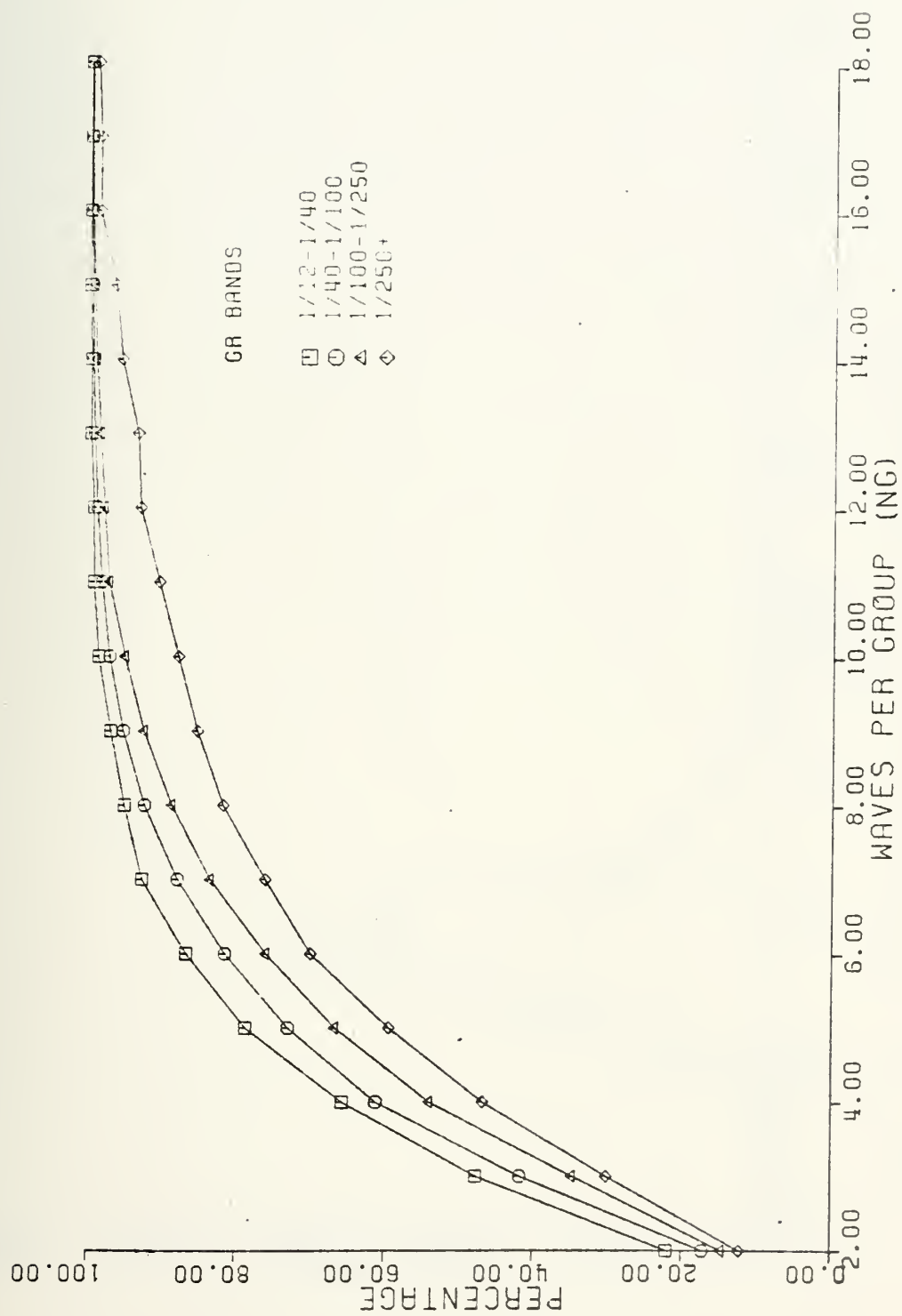


Figure 15B. Cumulative percentage distribution of  $N_G$  for  $G_R$  wave group bands.



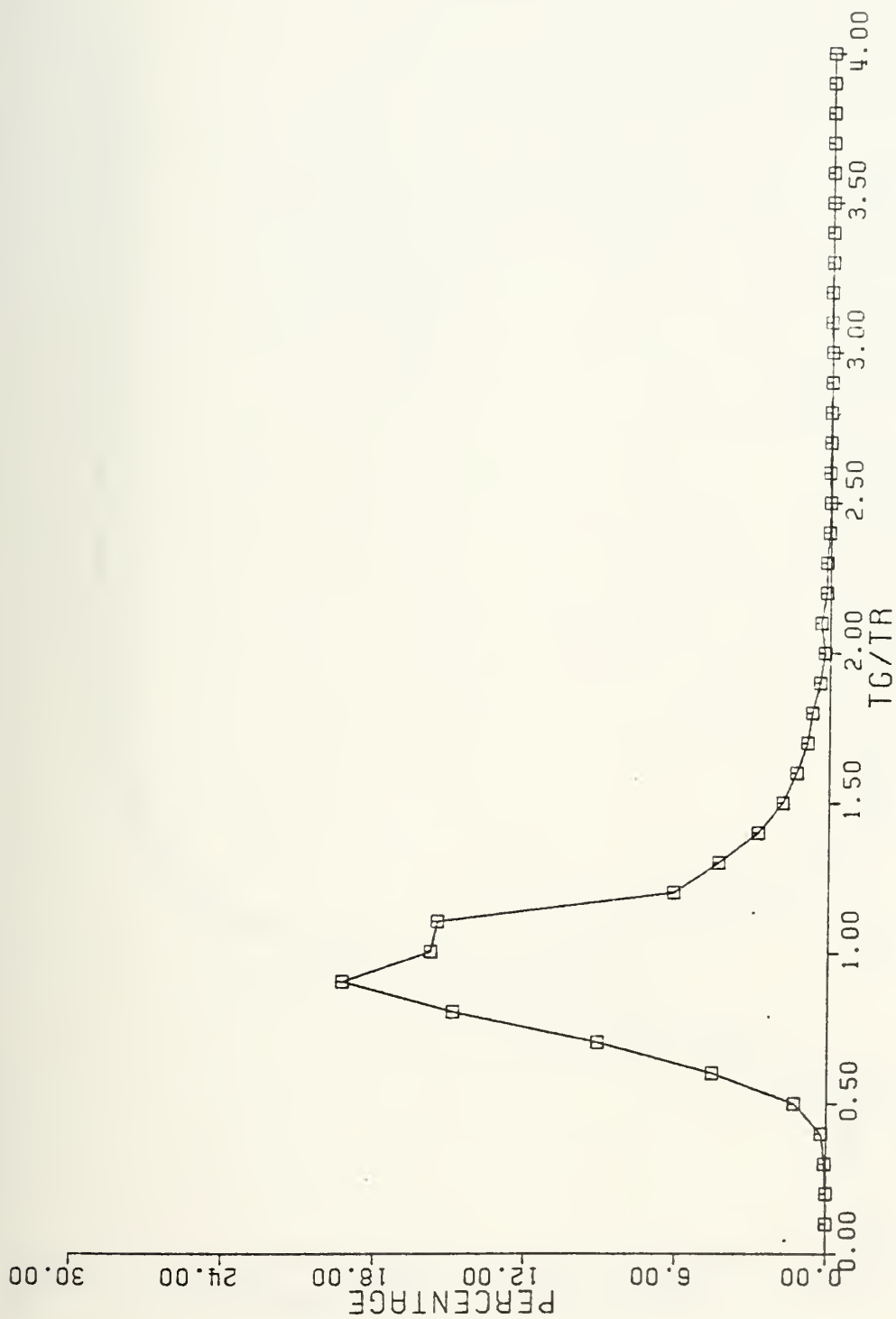


Figure 16A. Histogram of  $T_G/T_R$  for all wave groups.





Figure 16B. Cumulative percentage distribution of  $T_G/T_R$  for all wave groups.





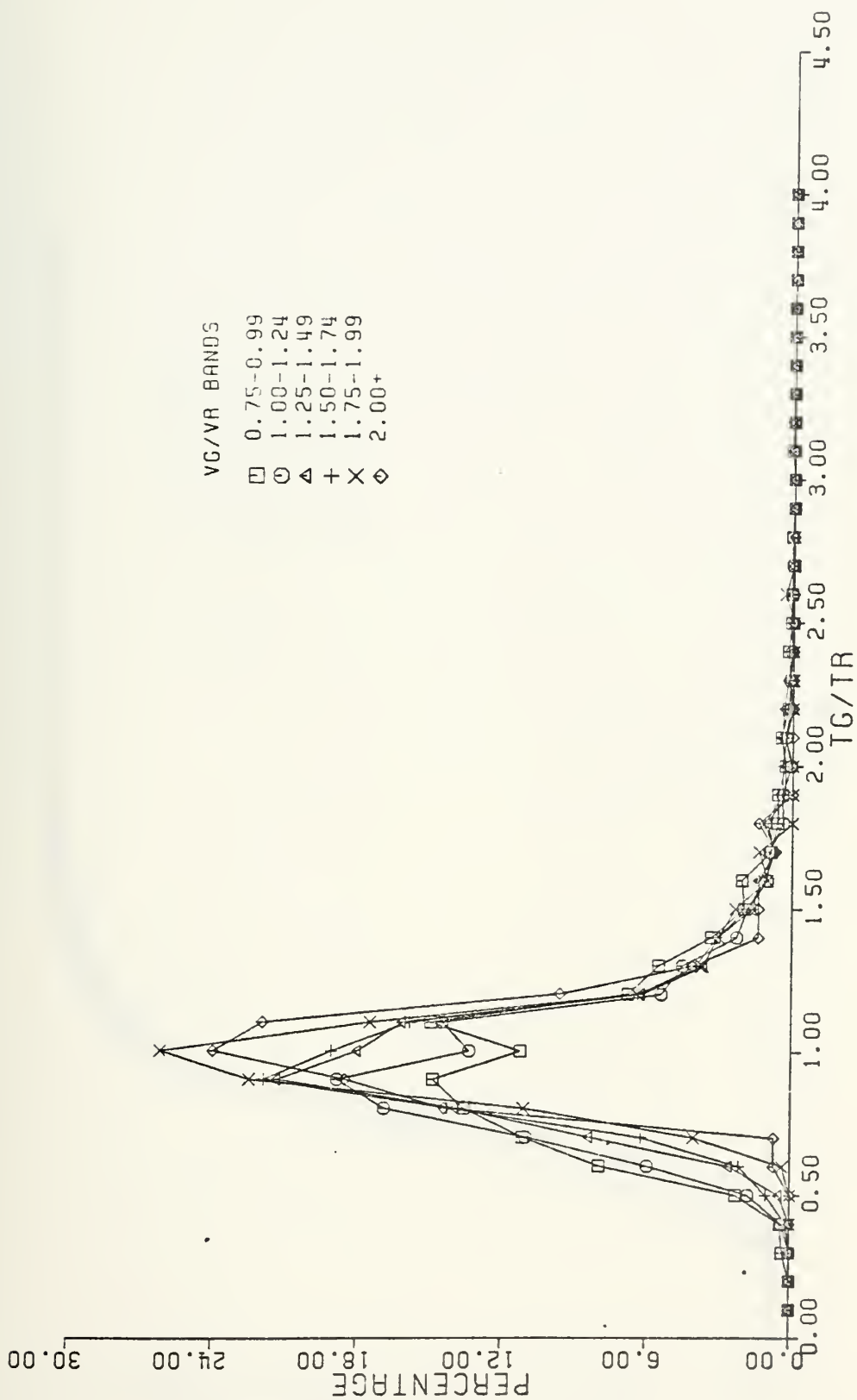


Figure 17A. Histogram of  $T_G/T_R$  for  $V_G/V_R$  wave group bands.



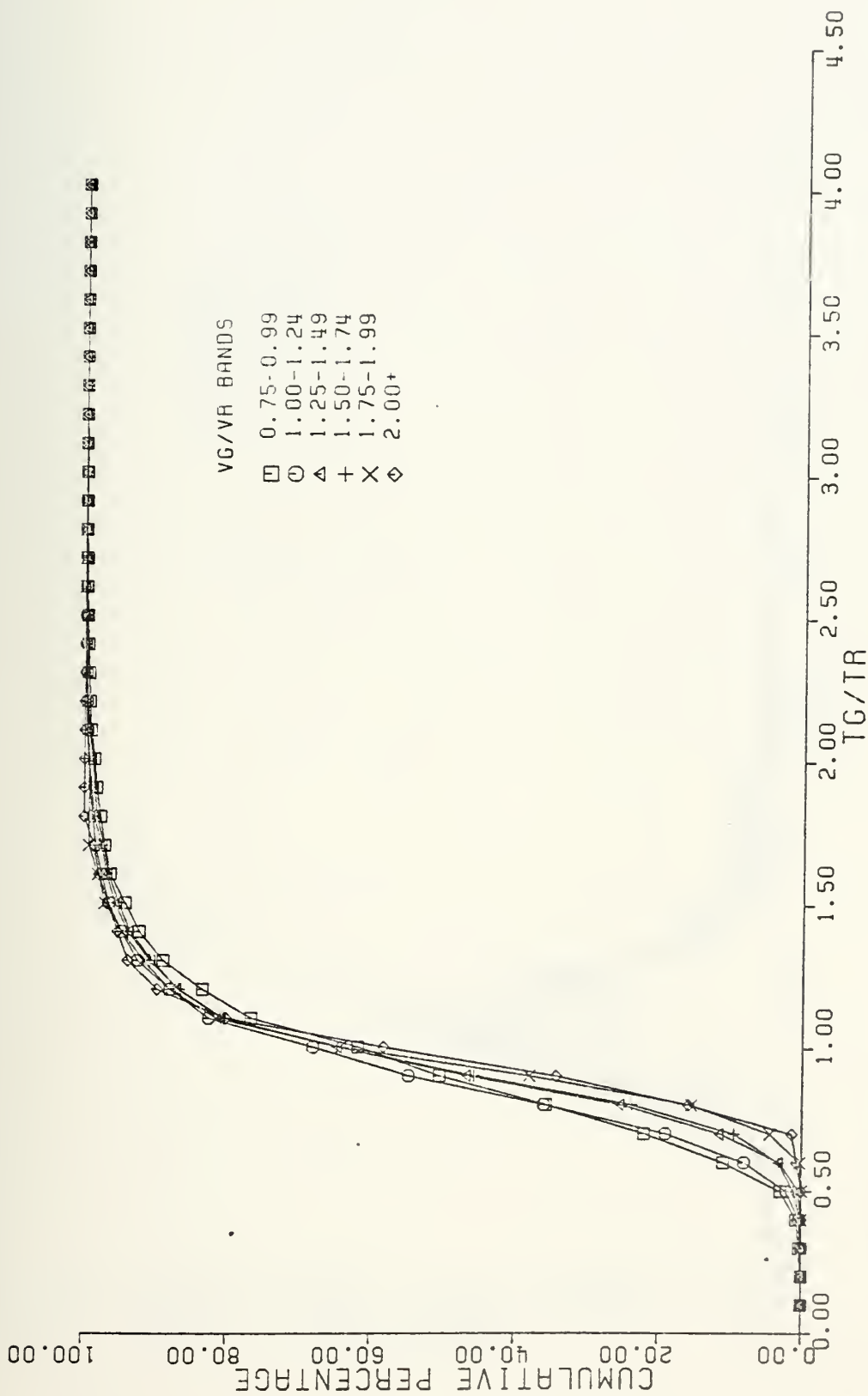


Figure 17B. Cumulative percentage distribution of  $T_G/T_R$  for  $V_G/V_R$  wave group bands.



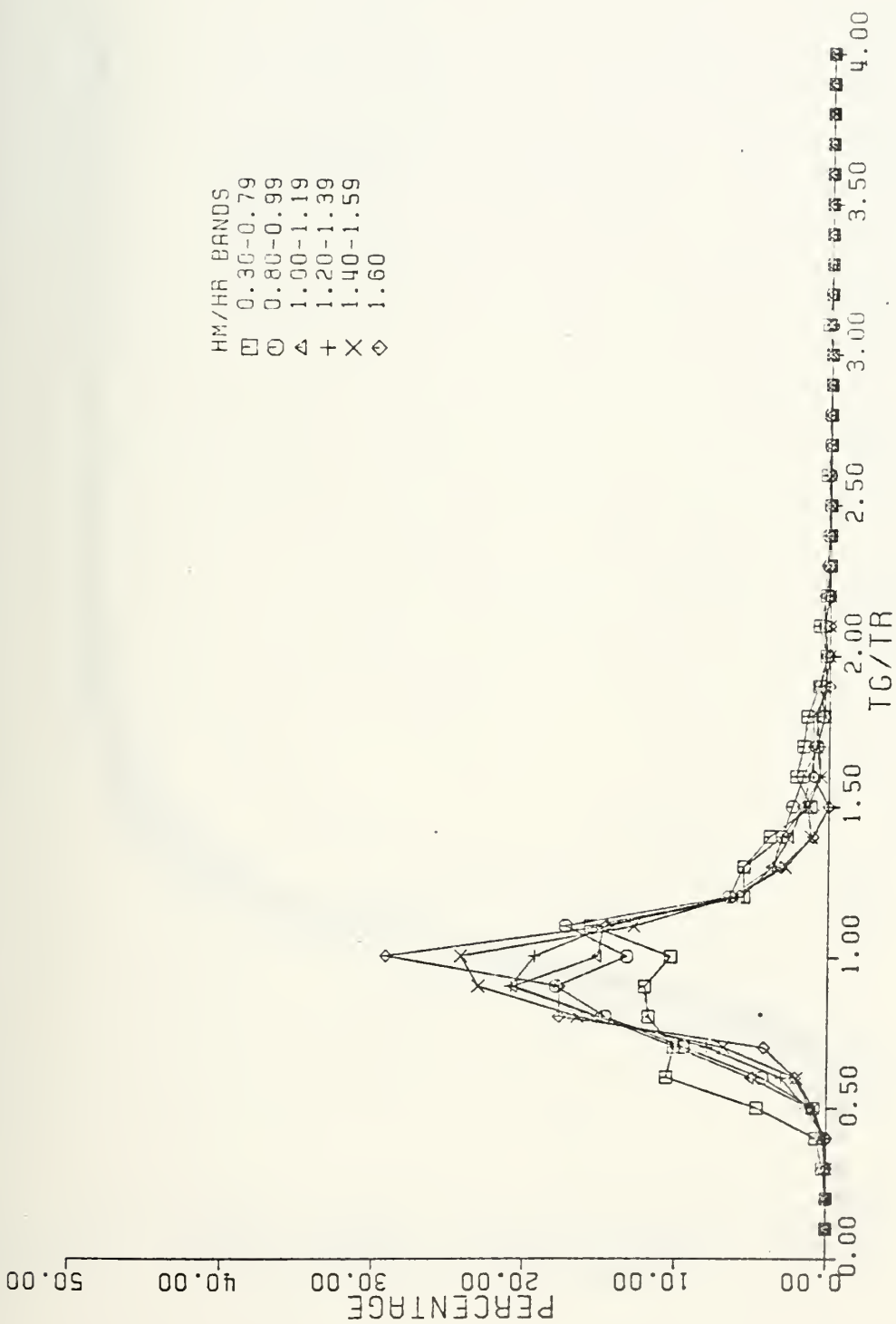


Figure 18A. Histogram of  $T_G/T_R$  for  $H_M/H_R$  wave group bands.



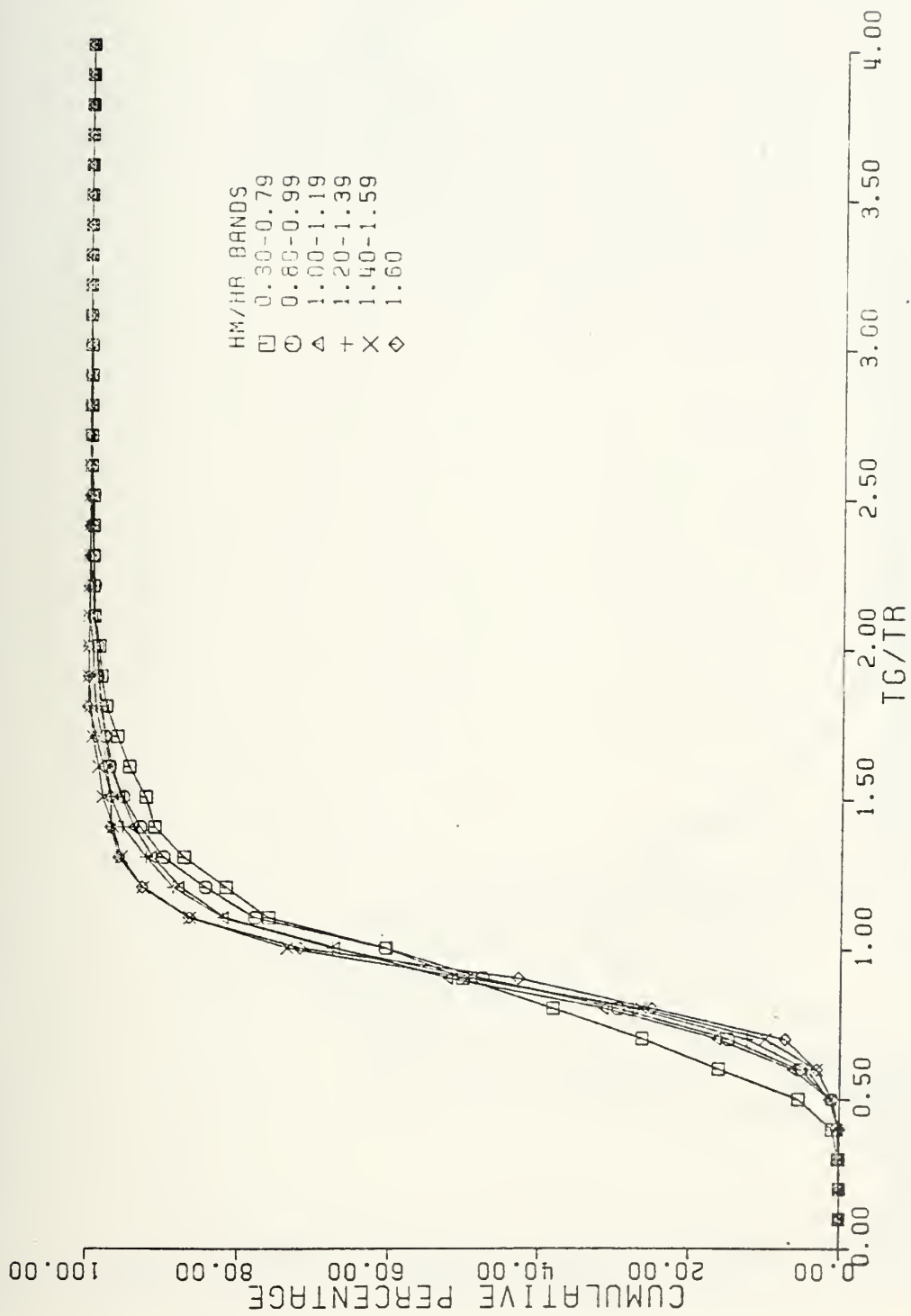


Figure 18B. Cumulative percentage distribution of  $T_G/T_R$  for  $H_M/H_R$  wave group bands.





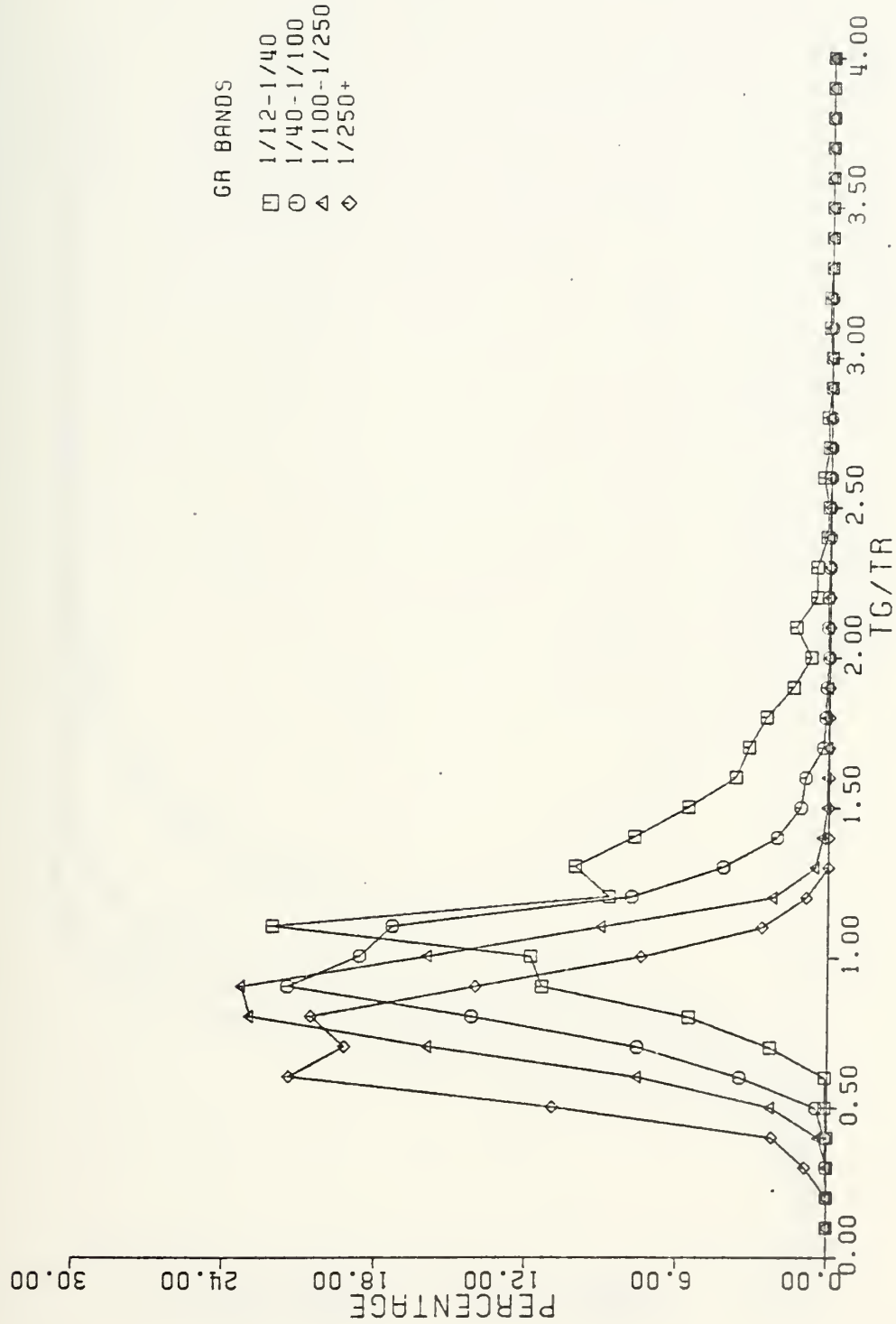


Figure 19A. Histogram of  $T_C/T_R$  for  $G_R$  wave group bands.



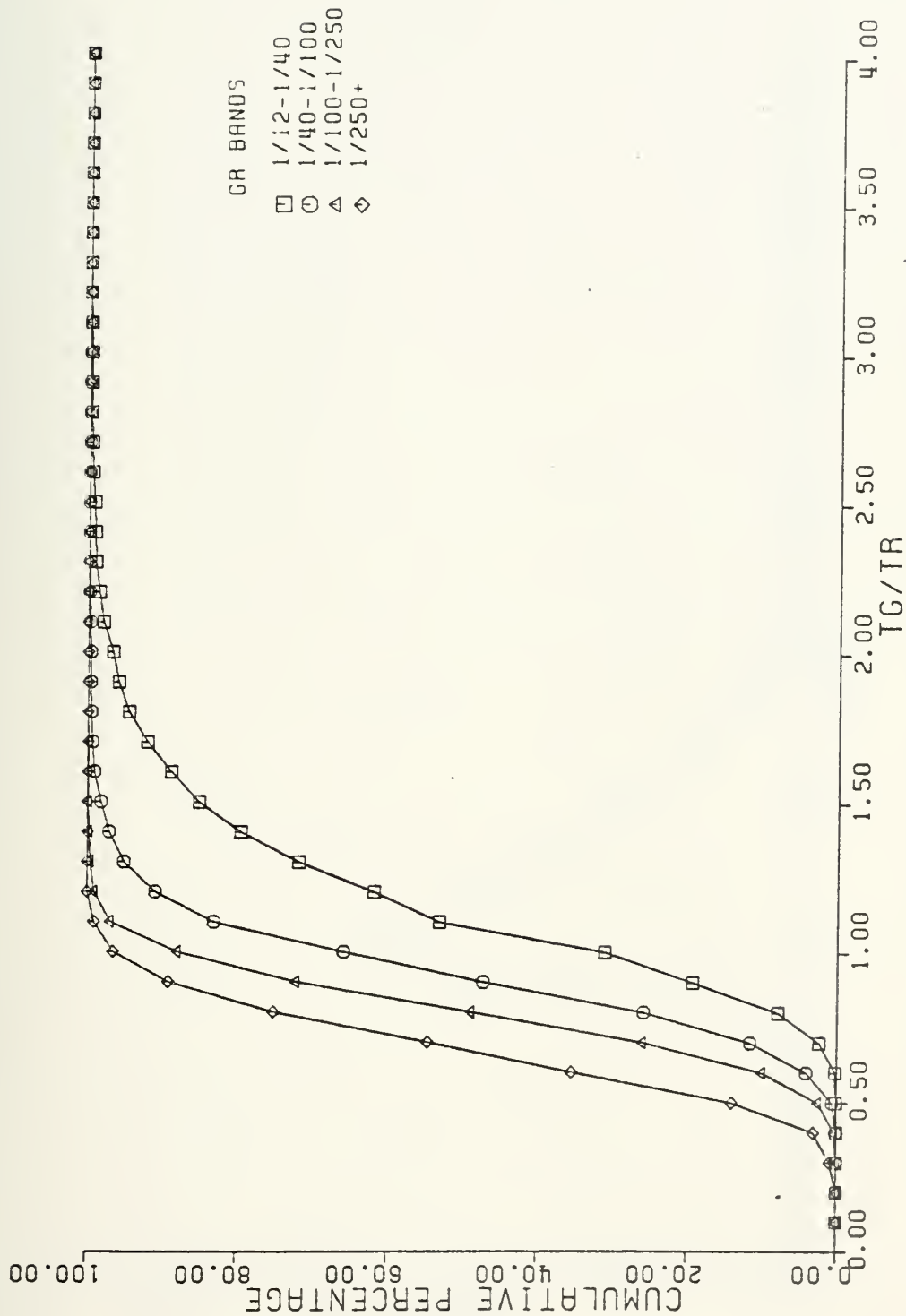


Figure 19B. Cumulative percentage distribution of  $T_G/T_R$  for  $G_R$  wave group bands.



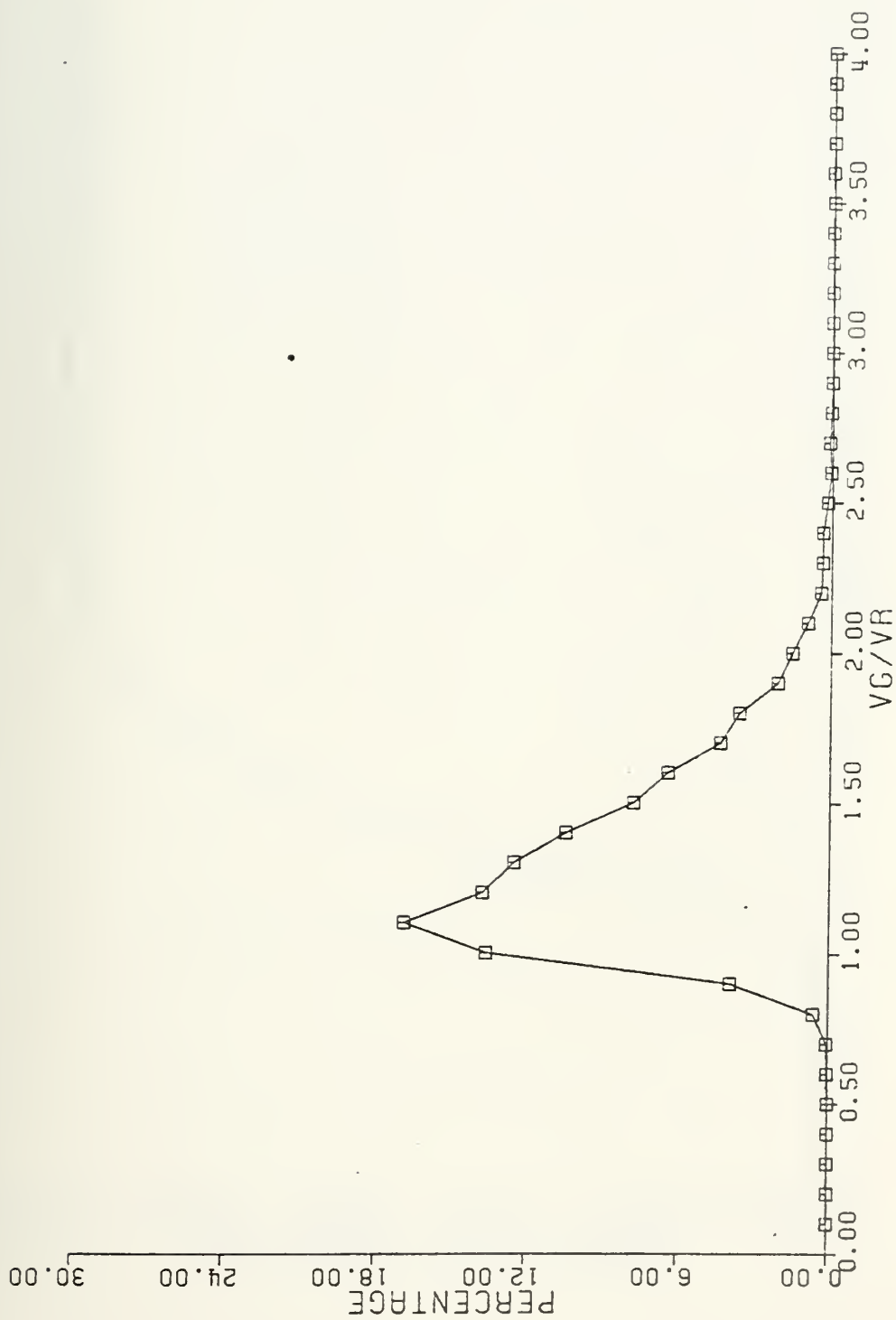


Figure 20A. Histogram of  $V_G/V_R$  for all wave groups.





Figure 20B. Cumulative percentage distribution of  $V_G/V_R$  for all wave groups.





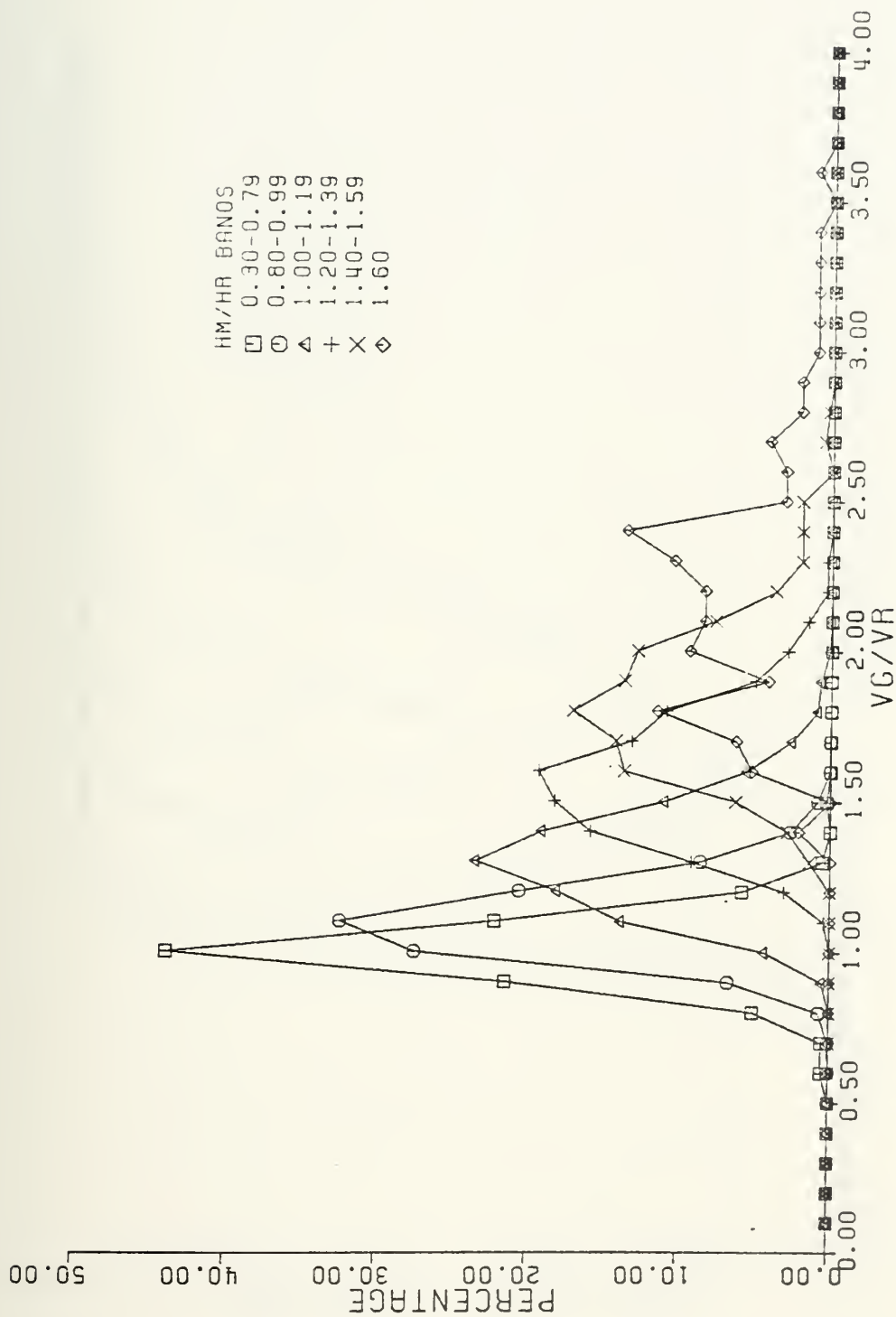


Figure 21A. Histogram of  $V_G/V_R$  for  $H_M/H_R$  wave group bands.



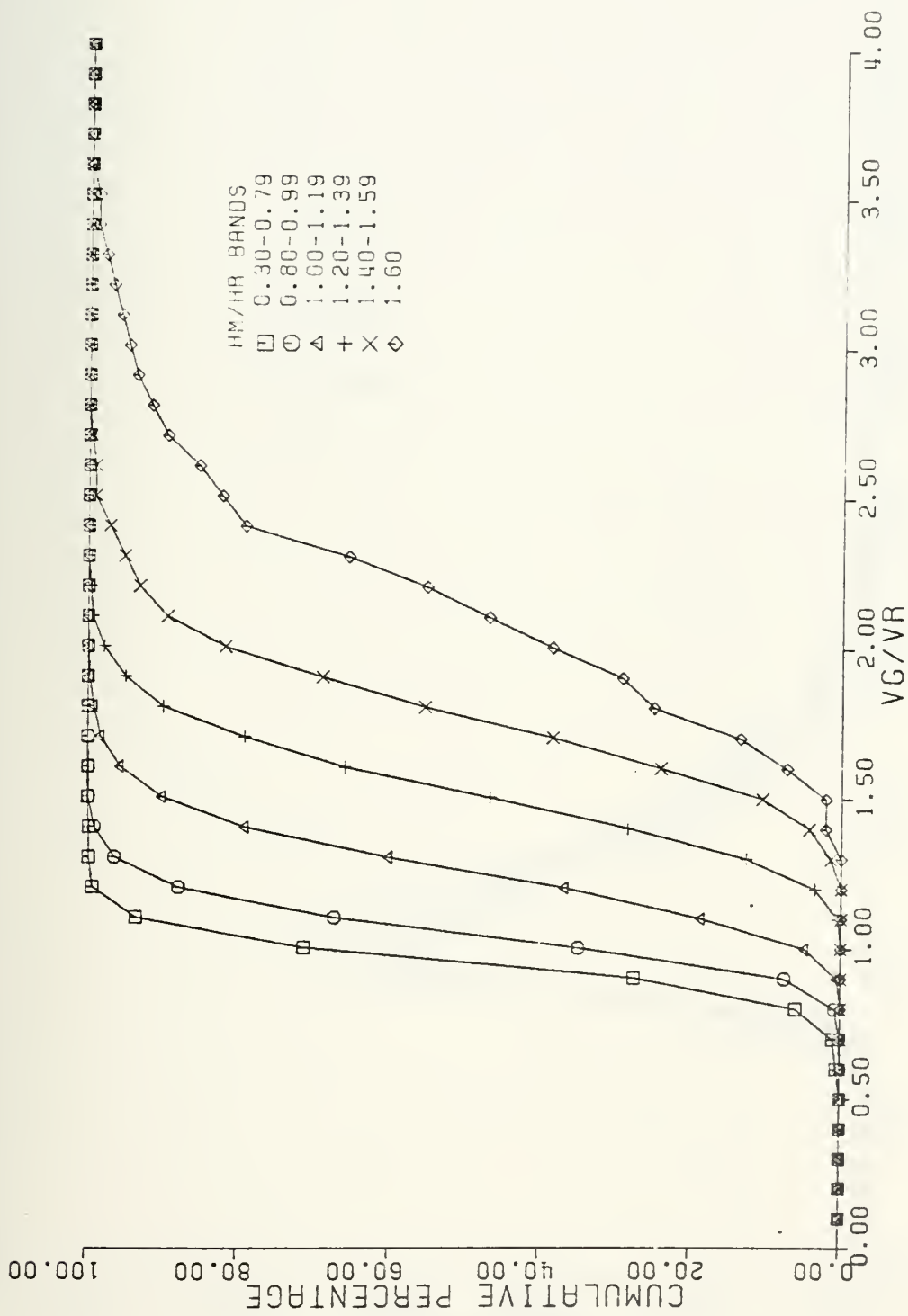


Figure 21B. Cumulative percentage distribution of  $V_G/V_R$  for  $H_M/H_R$  wave group bands.



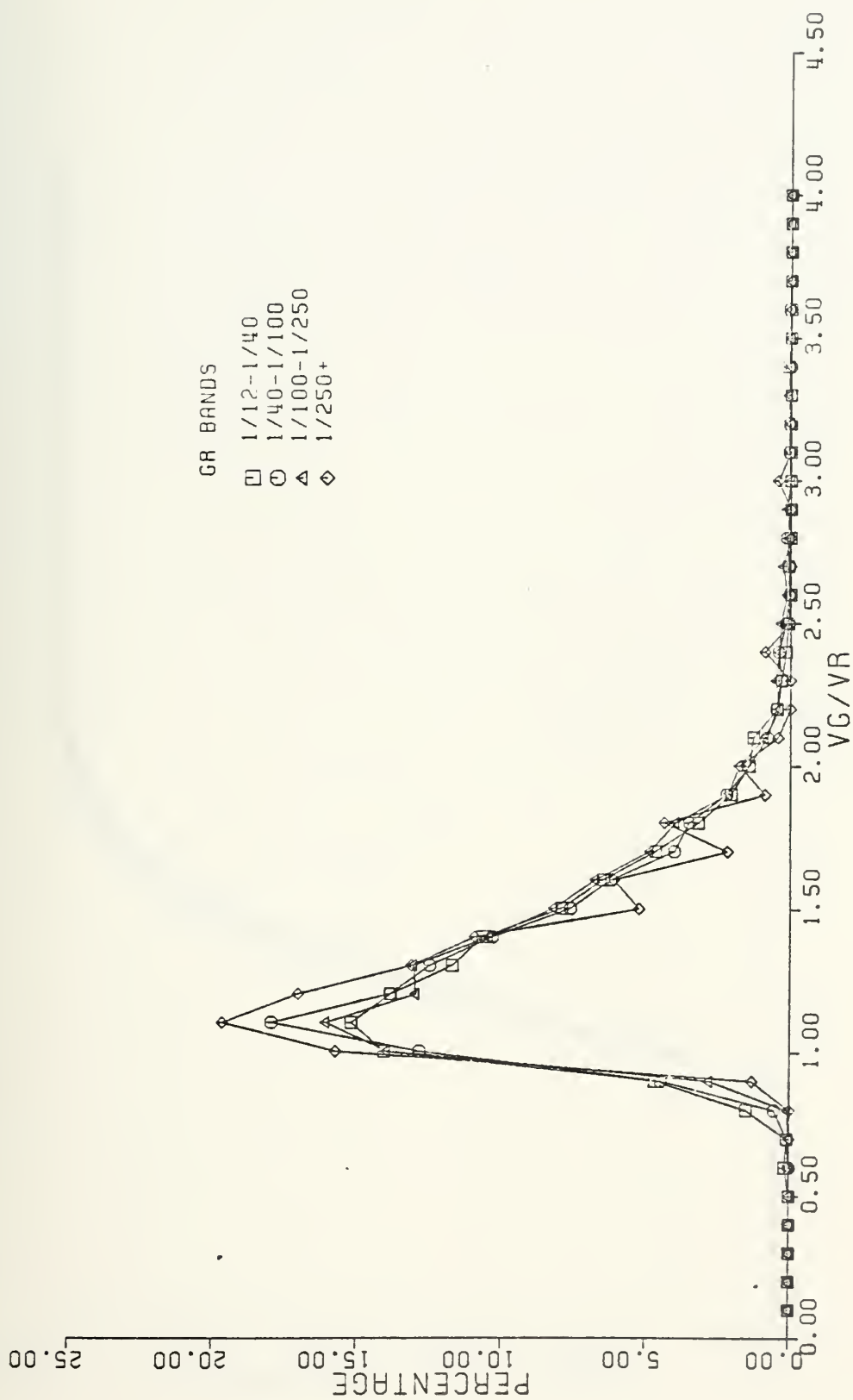


Figure 22A. Histogram of  $V_G/V_R$  for  $G_R$  wave group bands.





Figure 22B. Cumulative percentage distribution of  $V_G/V_R$  for  $G_R$  wave group bands.





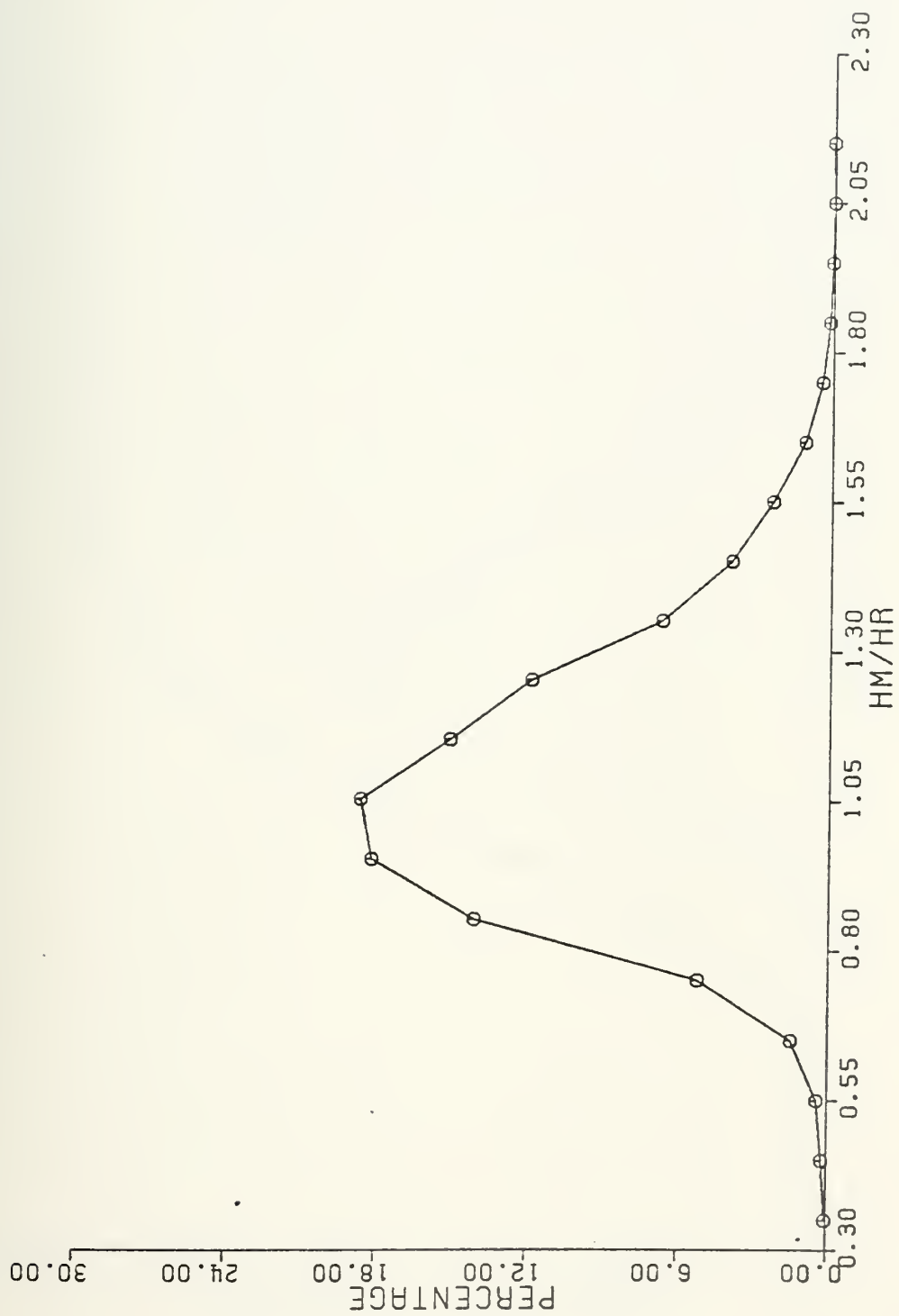


Figure 23A. Histogram of  $H_M/H_R$  for all wave groups.



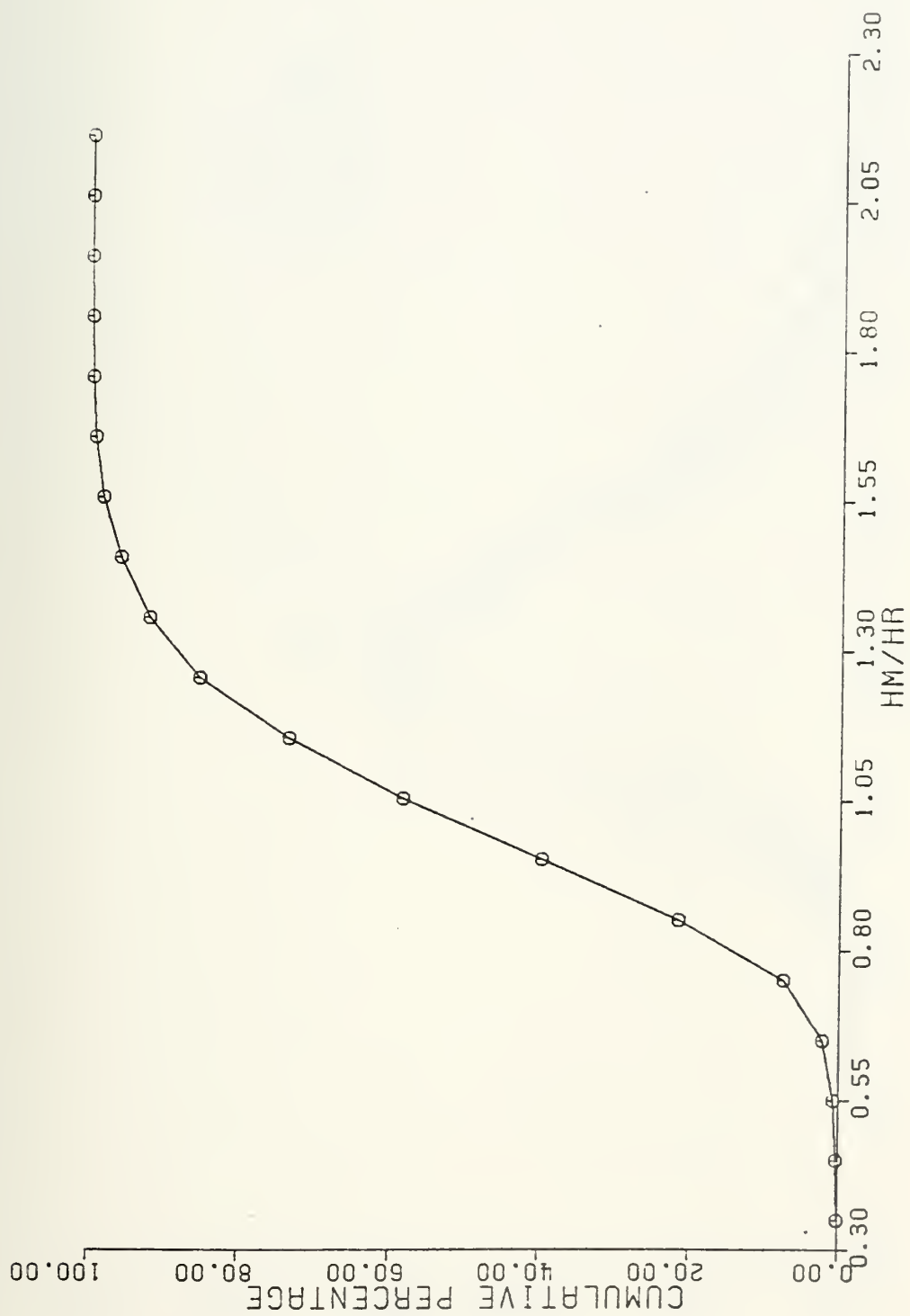


Figure 23B. Cumulative percentage distribution of  $H_M/H_R$  for all wave groups.



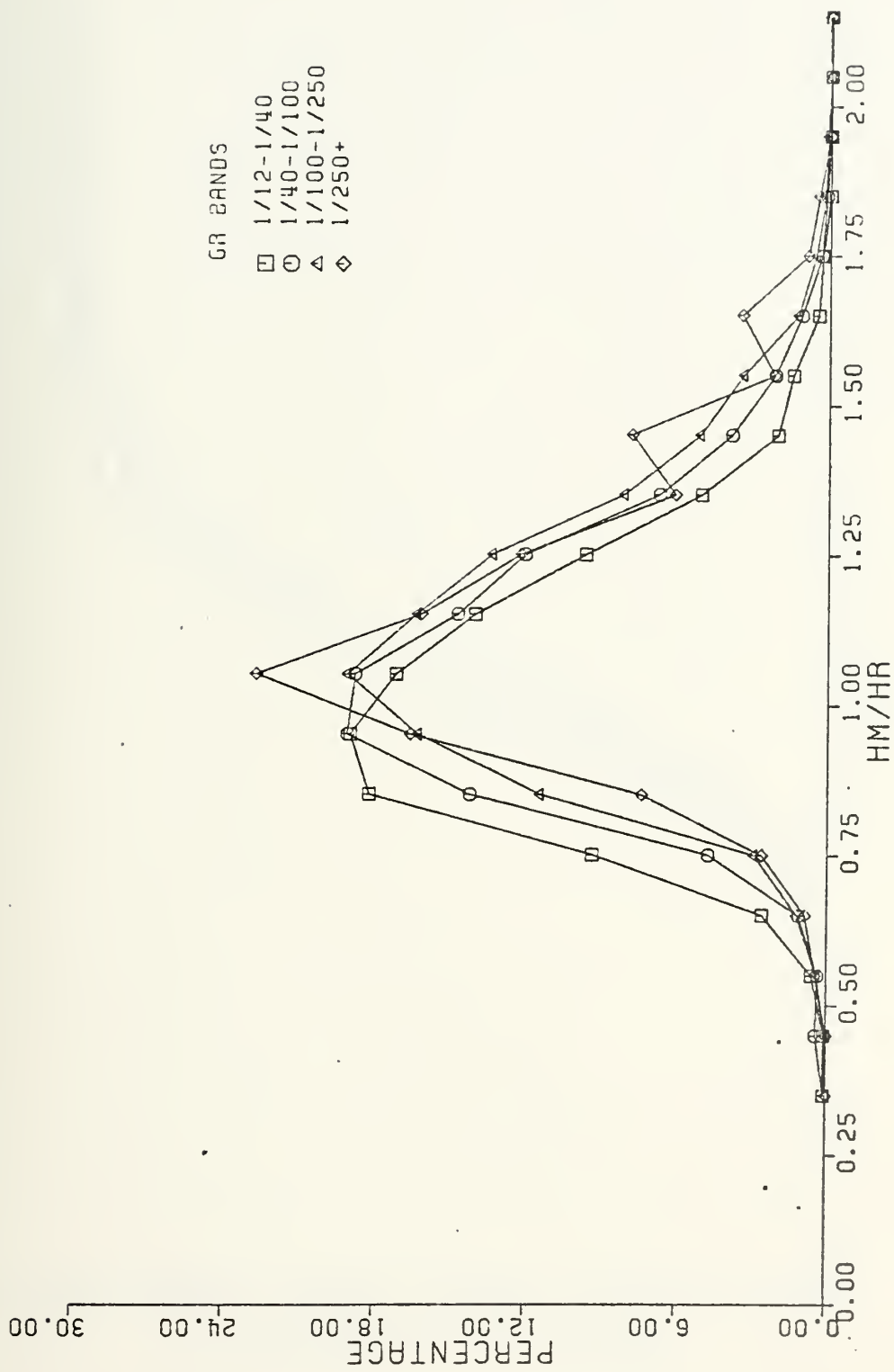


Figure 24A. Histogram of  $H_M/H_R$  for  $G_R$  wave group bands.



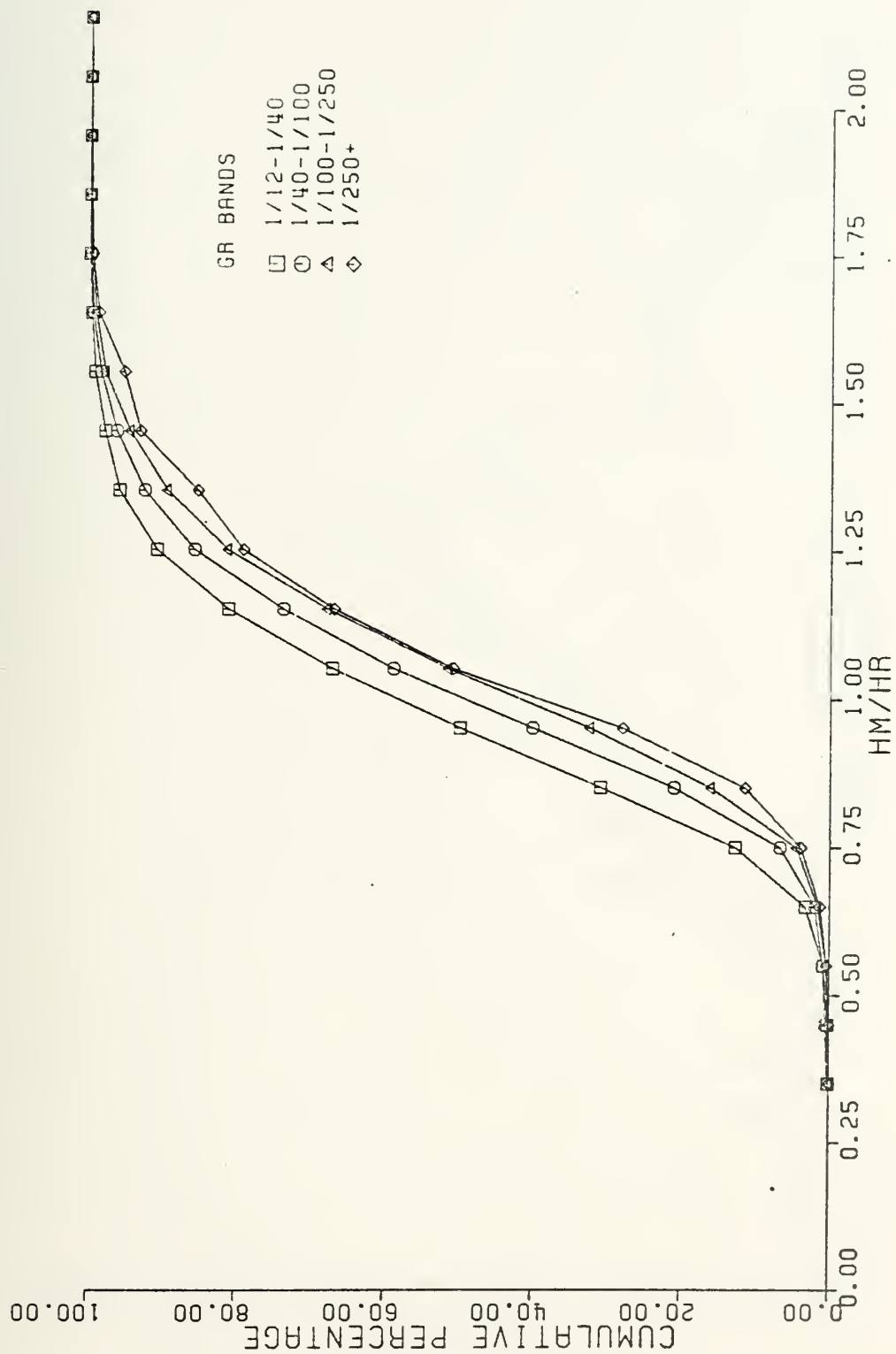


Figure 24B. Cumulative percentage distribution of  $H_M/H_R$  for  $G_R$  wave group bands.





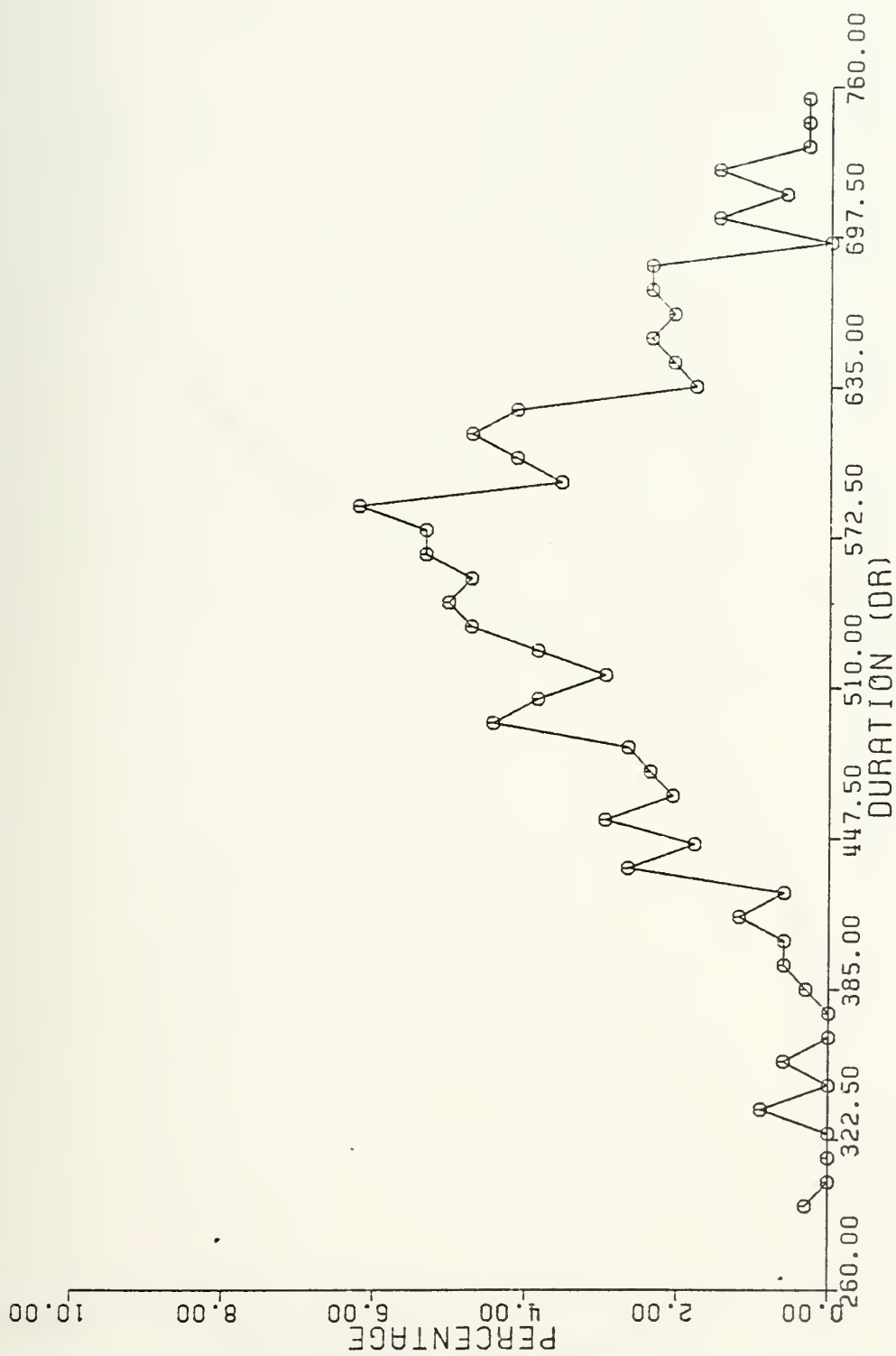


Figure 25A. Histogram of  $D_R$  for all wave groups.



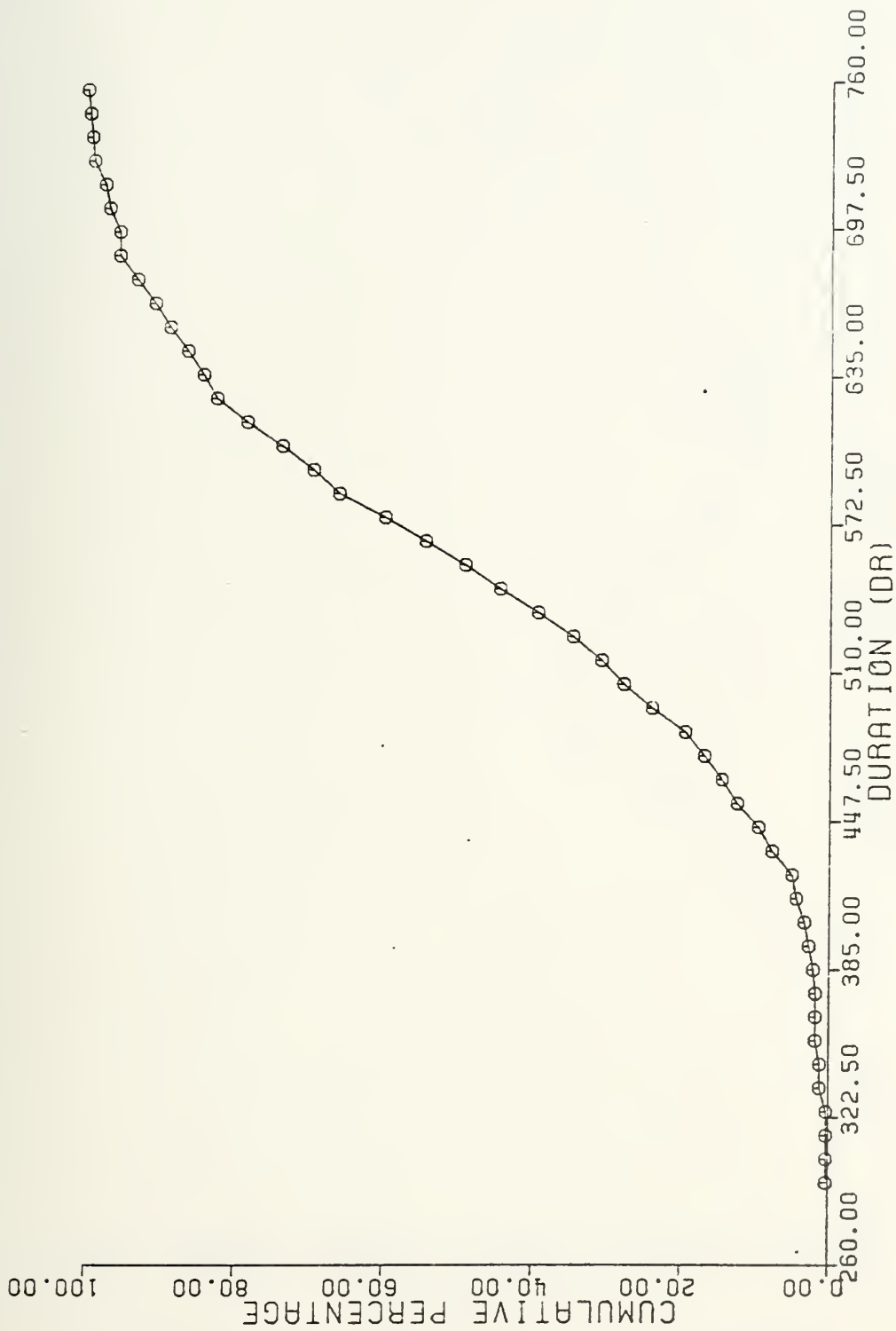


Figure 25B. Cumulative percentage distribution of  $D_R$  for all wave groups.



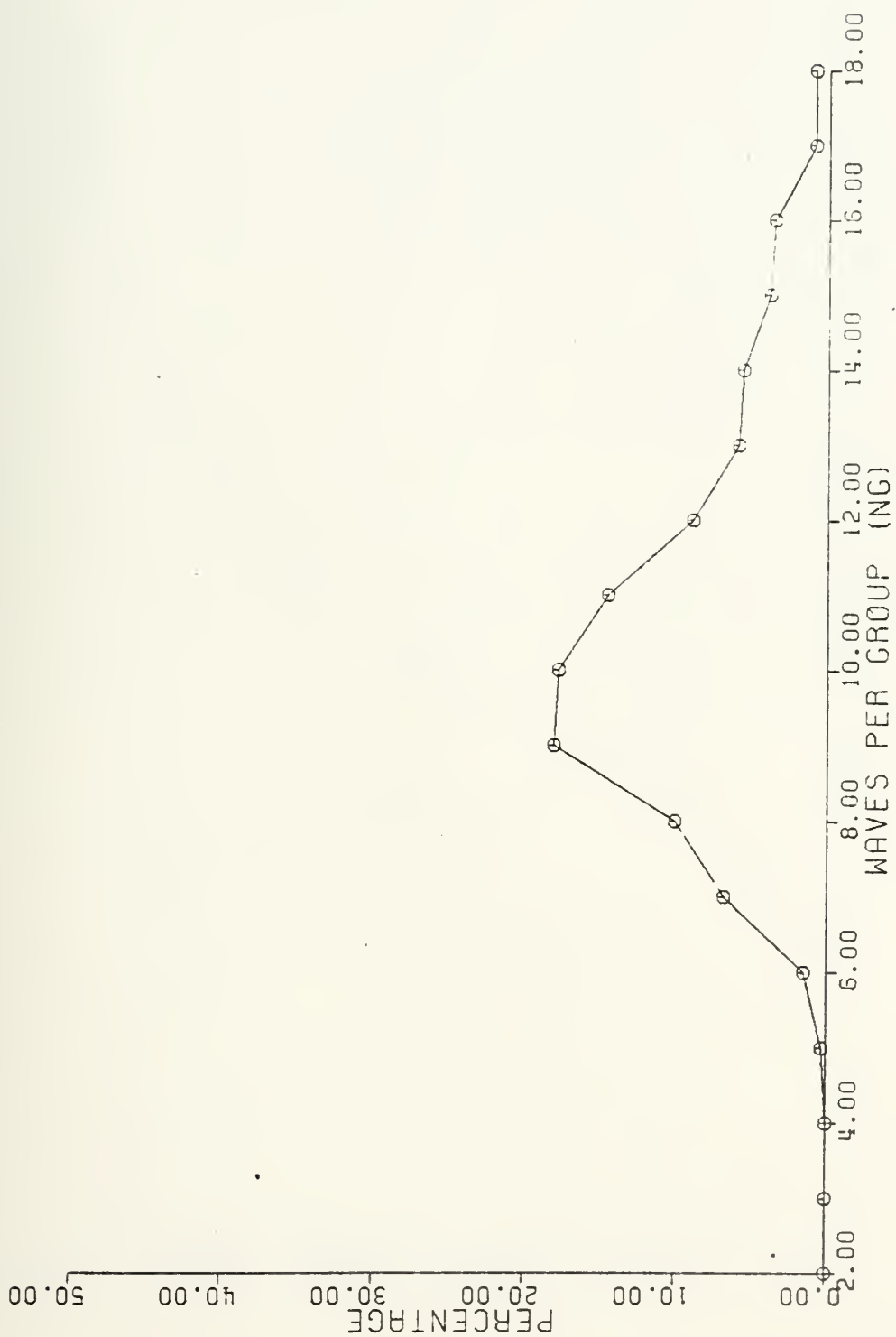


Figure 26A. Histogram of  $N_G$  for extreme wave groups: largest group  $N_G$  per wave record.



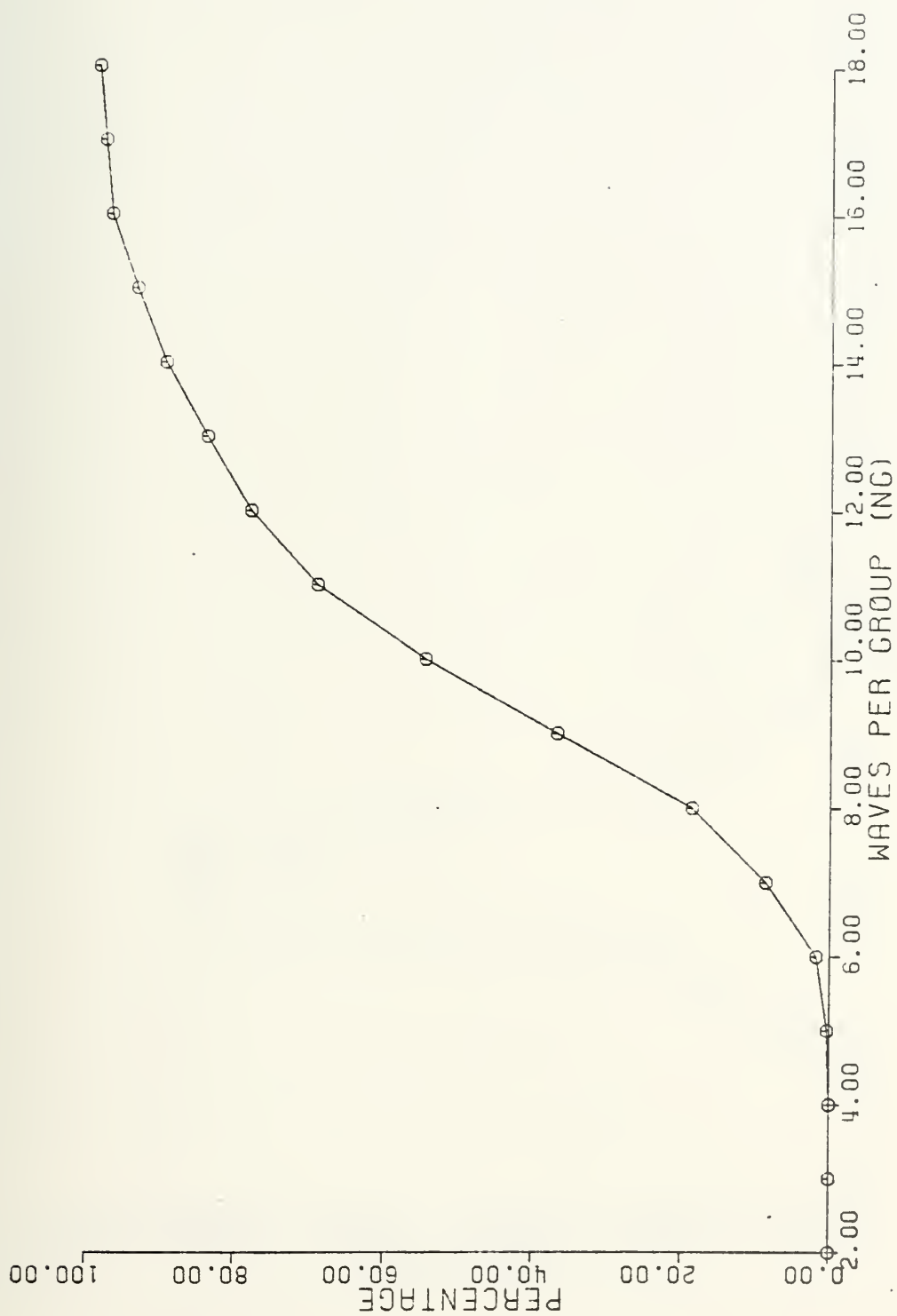


Figure 26B. Cumulative percentage distribution of  $N_G$  for extreme wave groups: largest group  $N_G$  per wave record.





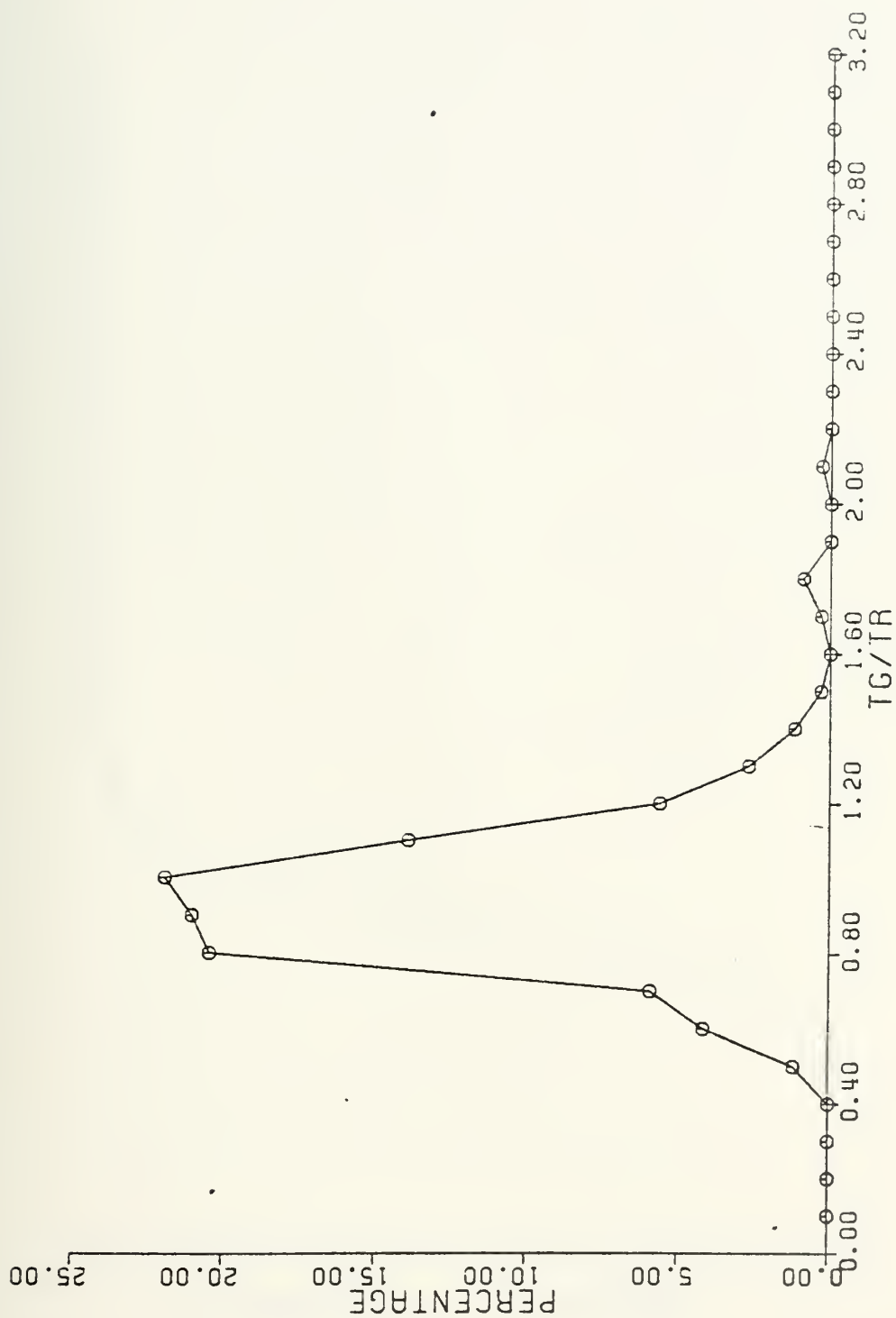


Figure 27A. Histogram of  $T_G/T_R$  for extreme wave groups: largest group  $N_G$  per wave record.



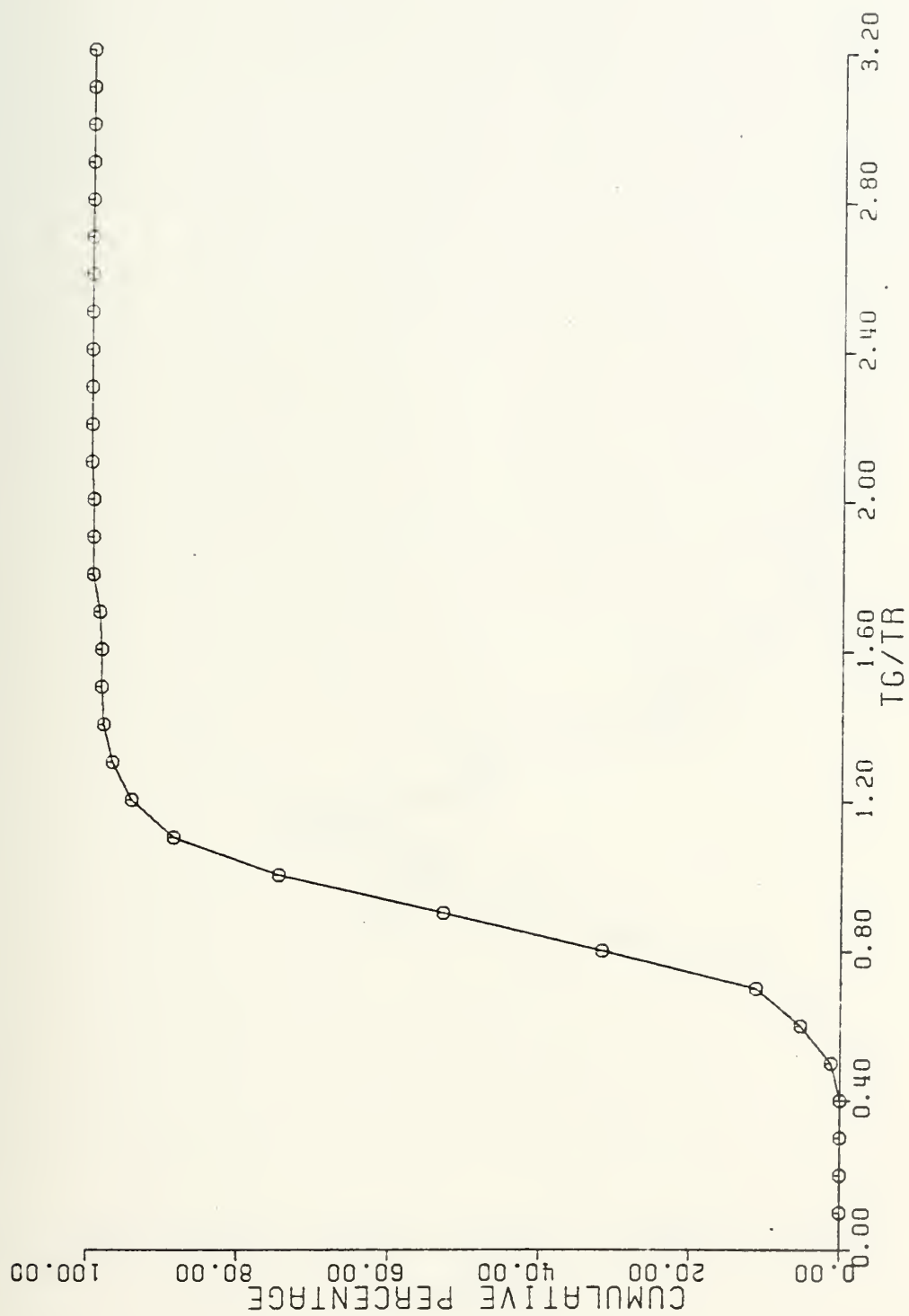


Figure 27B. Cumulative percentage distribution of  $T_G/T_R$  for extreme wave groups: largest group  $N_G$  per wave record.



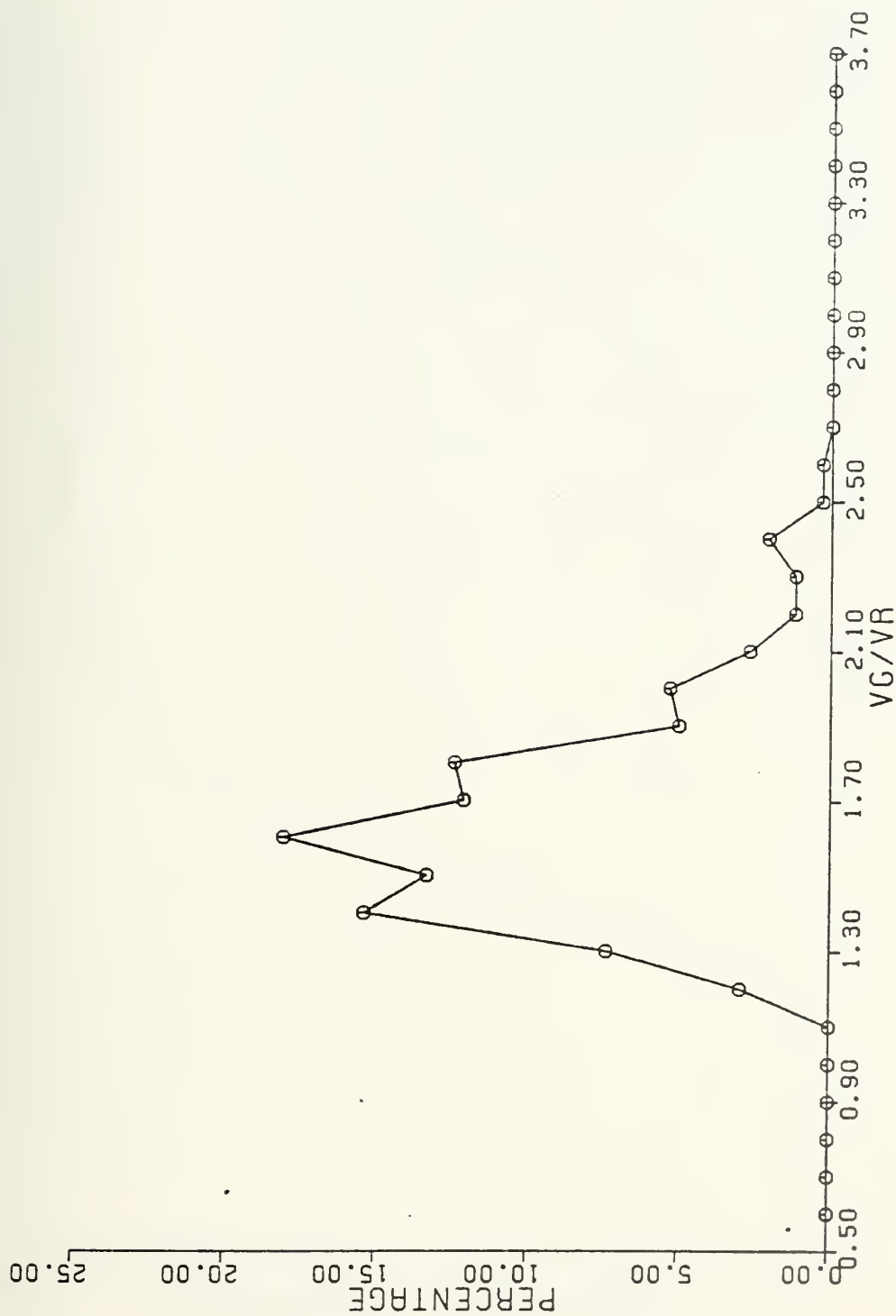


Figure 28A. Histogram of  $V_G/V_R$  for extreme wave groups; largest group  $N_G$  per wave record.



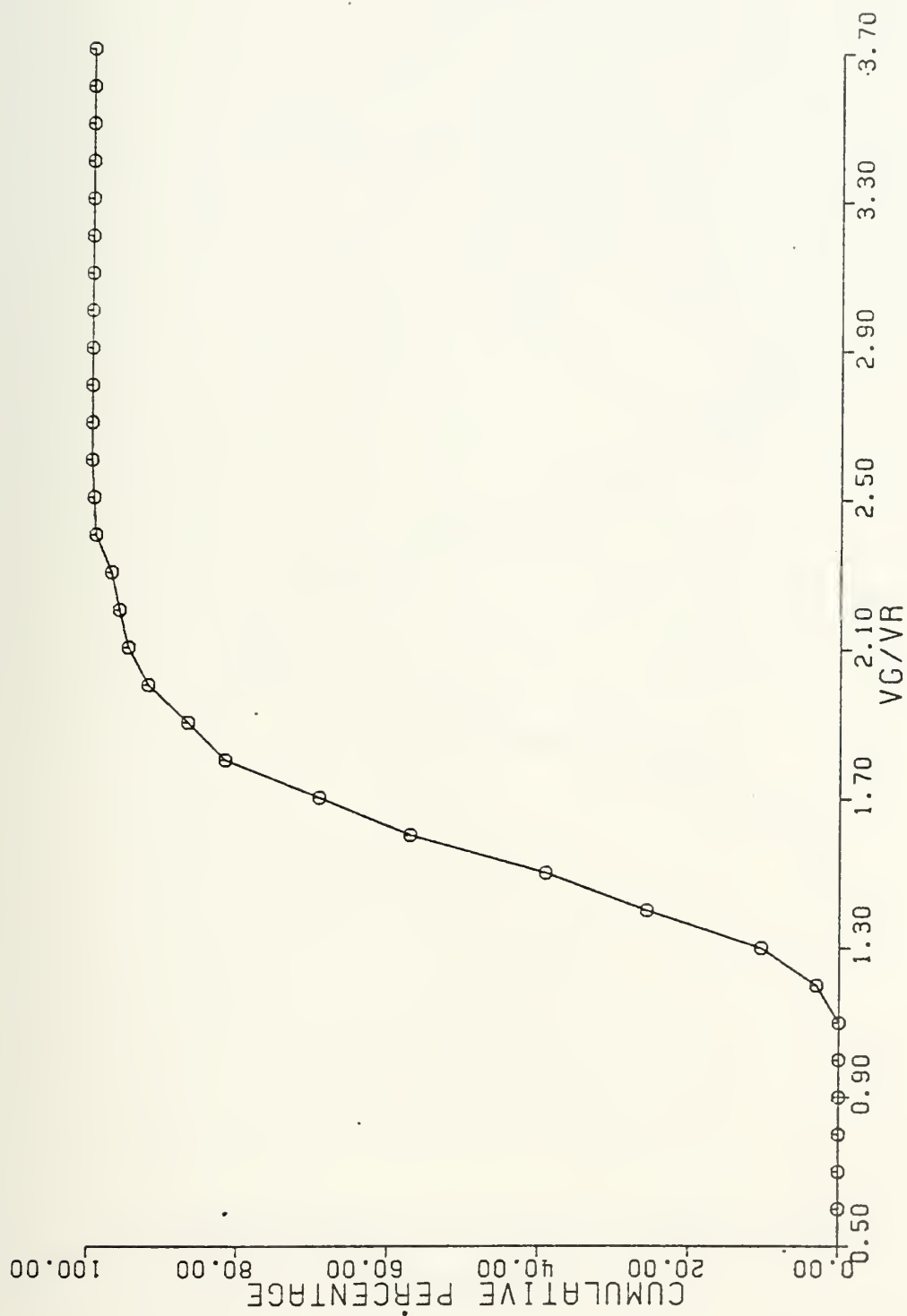


Figure 28B, Cumulative percentage distribution of  $V_G/V_R$  for extreme wave groups: largest group  $N_G$  per wave record.





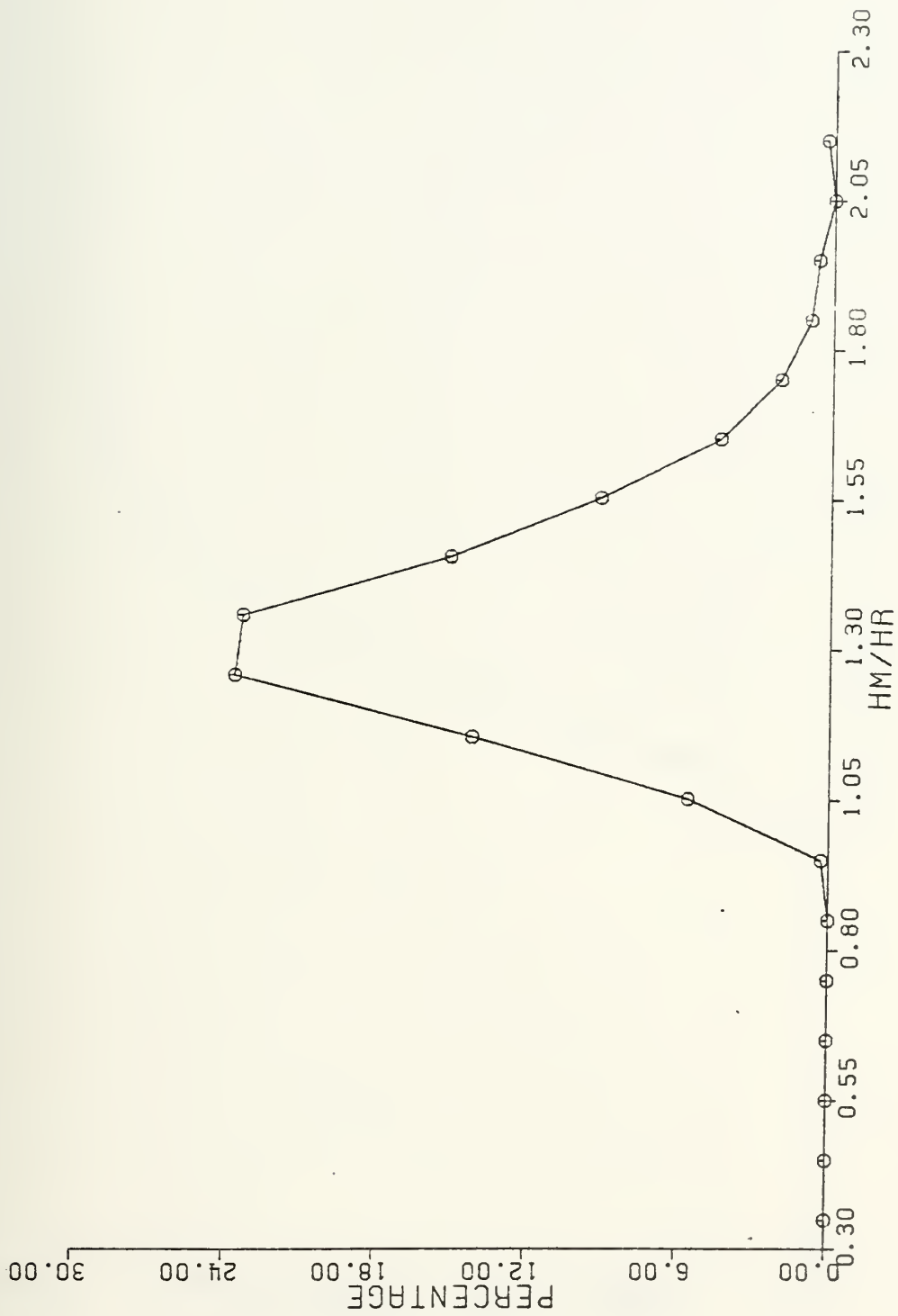


Figure 29A. Histogram of  $H_M/H_R$  for extreme wave groups: largest group  $N_G$  per wave record.



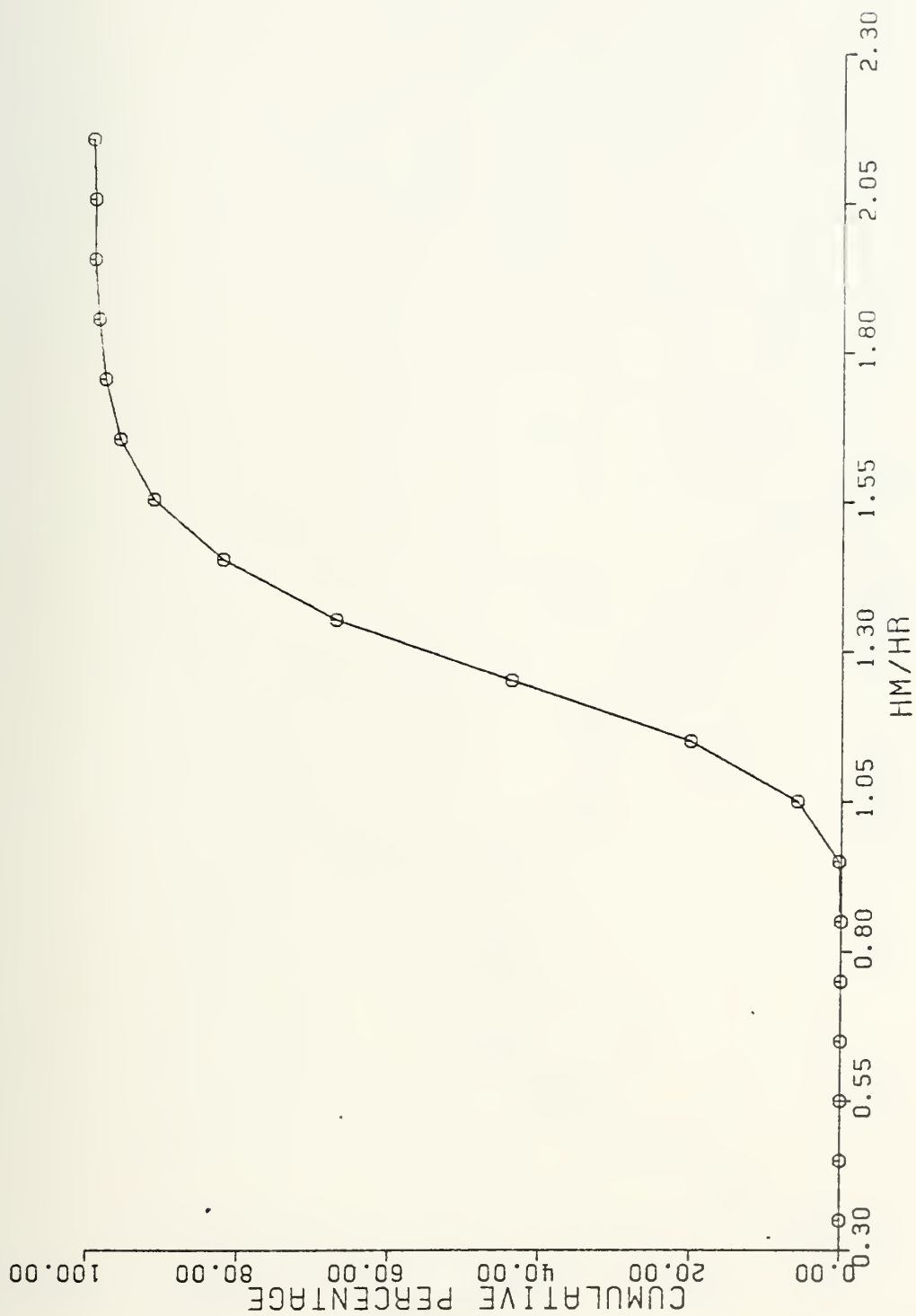


Figure 29B. Cumulative percentage distribution of  $H_M/H_R$  for extreme wave groups: largest group  $N_G$  per wave record.



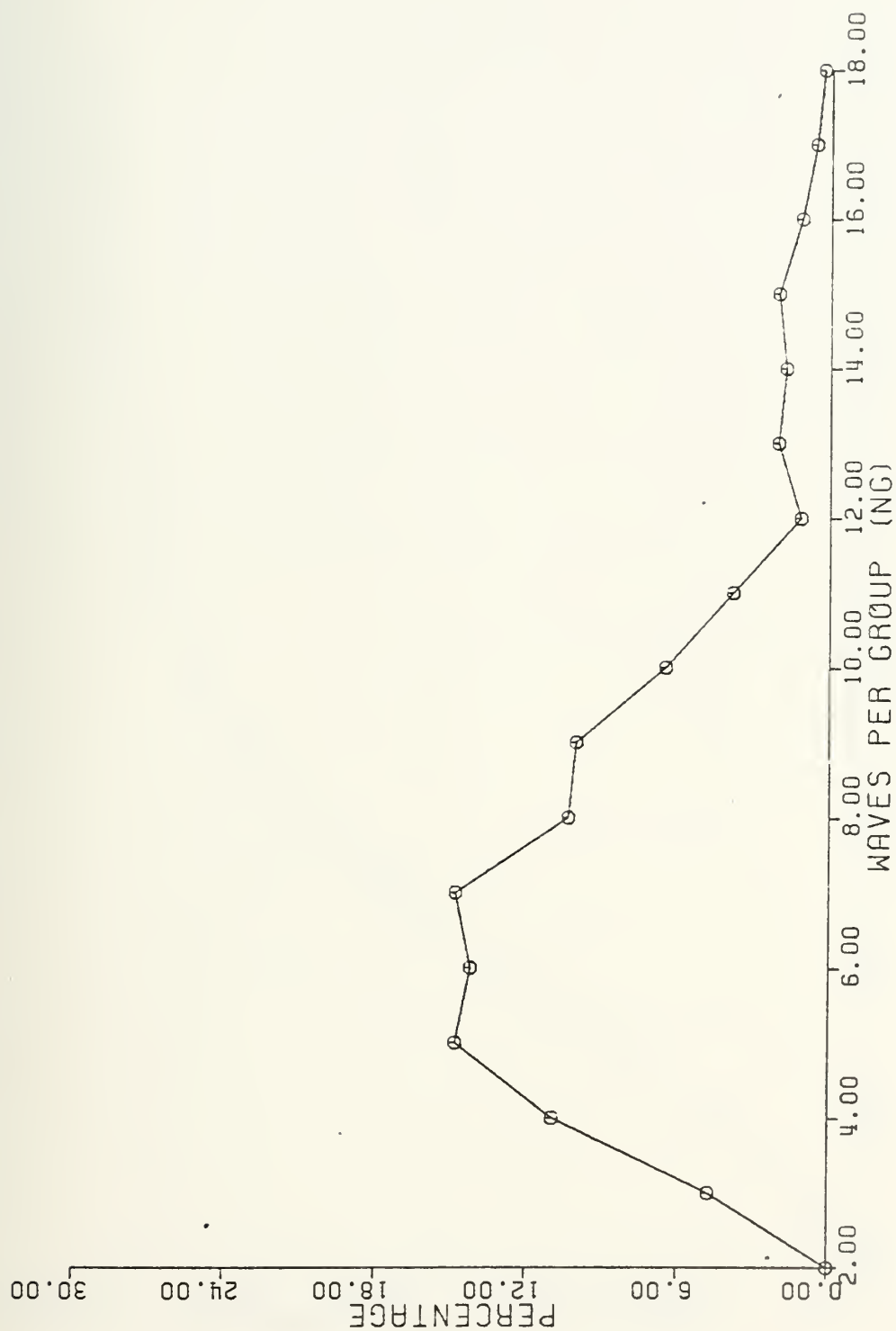


Figure 30A. Histogram of  $N_G$  for extreme wave groups: largest group  $V_G/V_R$  per wave record.



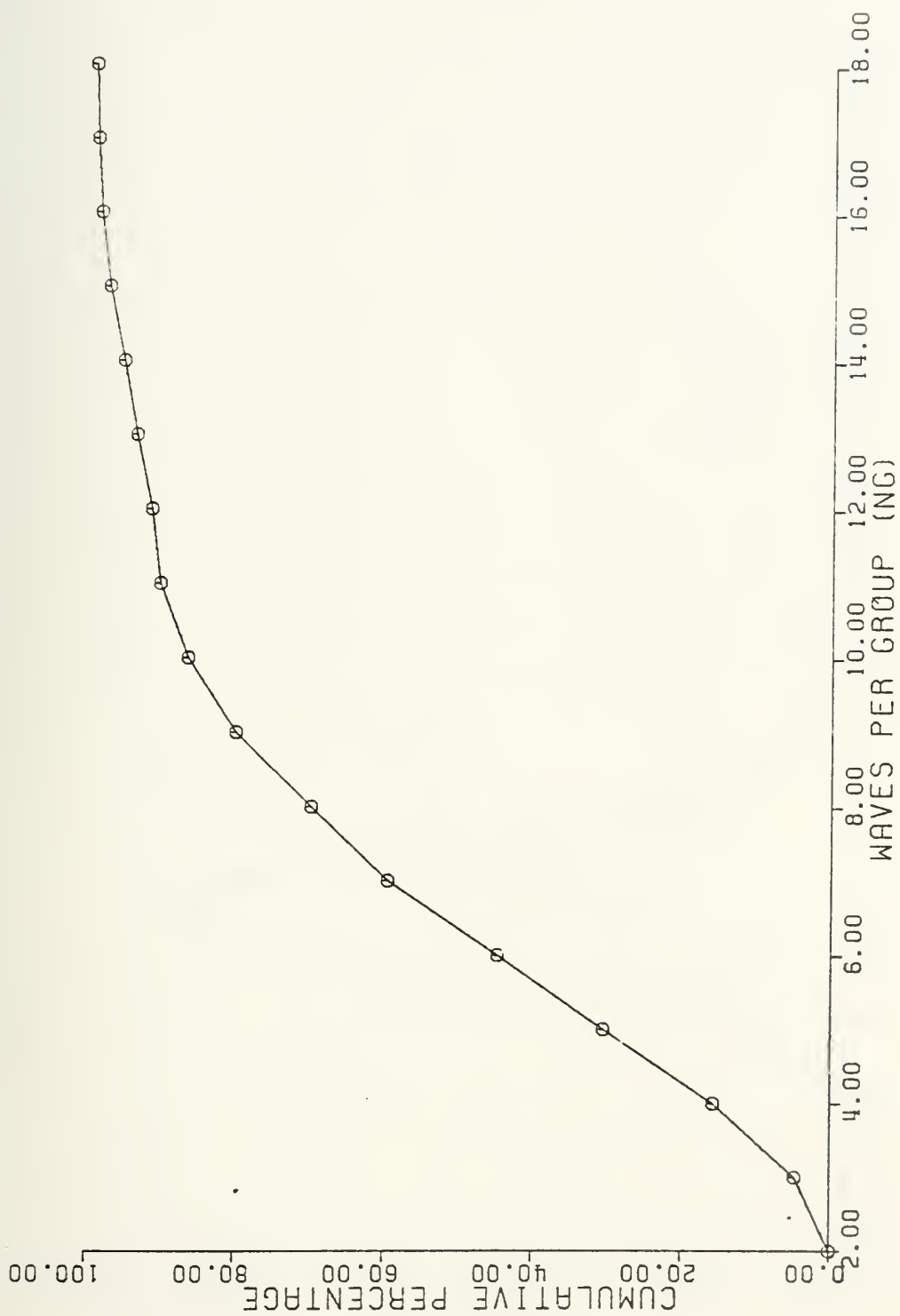


Figure 30B. Cumulative percentage distribution of  $N_G$  for extreme wave groups: largest group  $V_G/V_R$  per wave record.





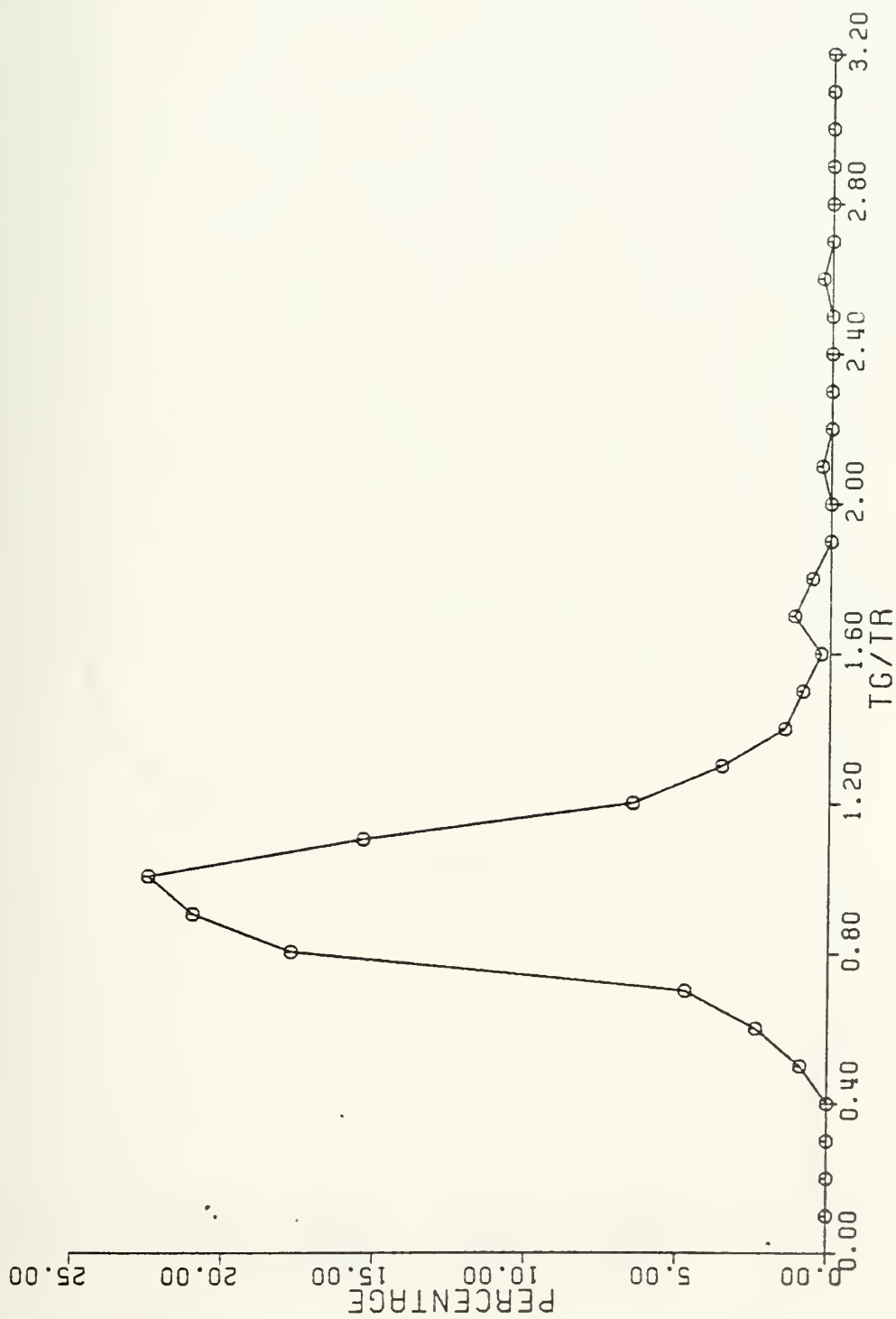


Figure 31A. Histogram of  $T_G/T_R$  for extreme wave groups: largest group  $V_G/V_R$  per wave record.



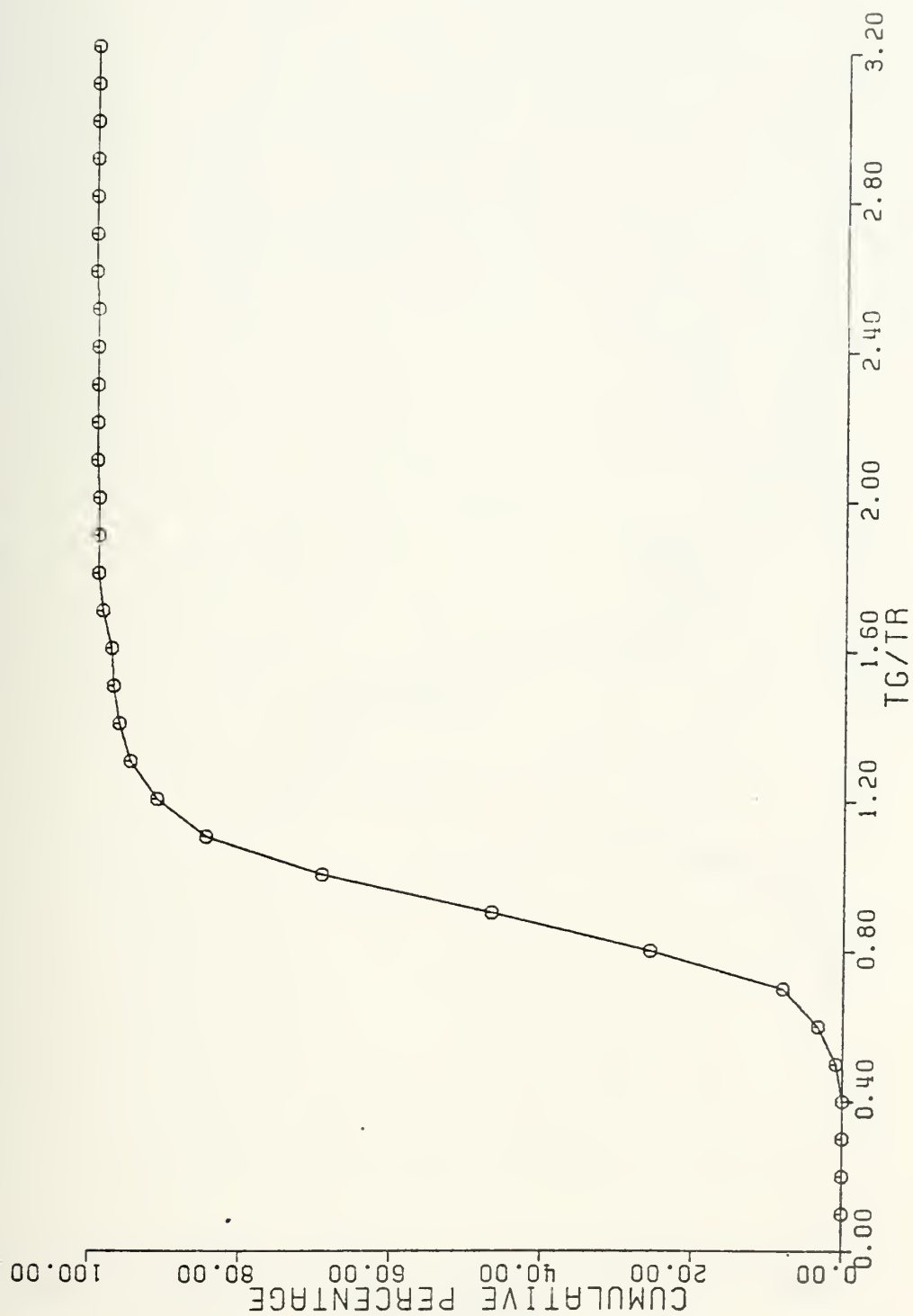


Figure 31B. Cumulative percentage distribution of  $T_G/T_R$  for extreme wave groups: largest group  $V_G/V_R$  per wave record.



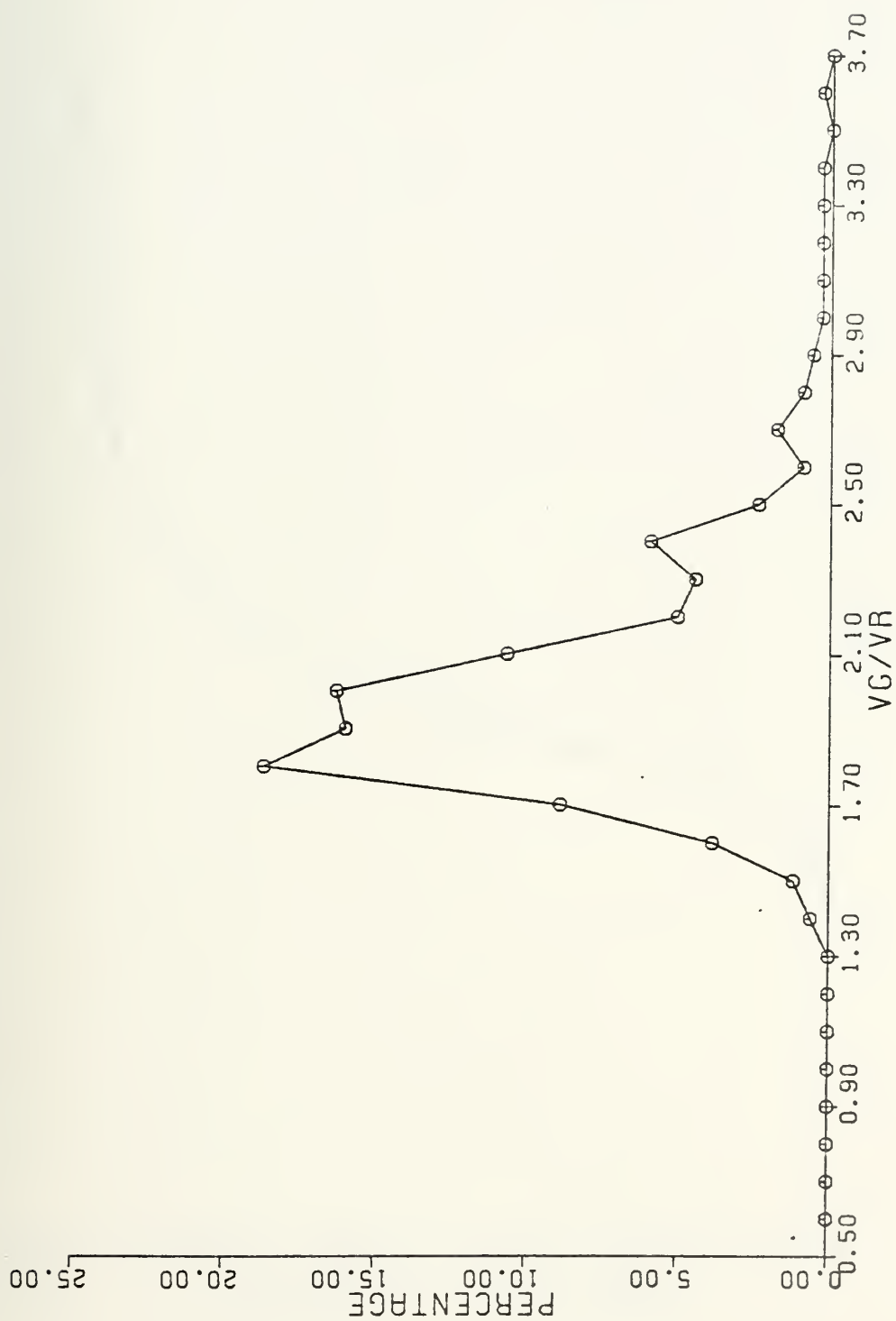


Figure 32A. Histogram of  $V_G/V_R$  for extreme wave groups; largest group  $V_G/V_R$  per wave record.



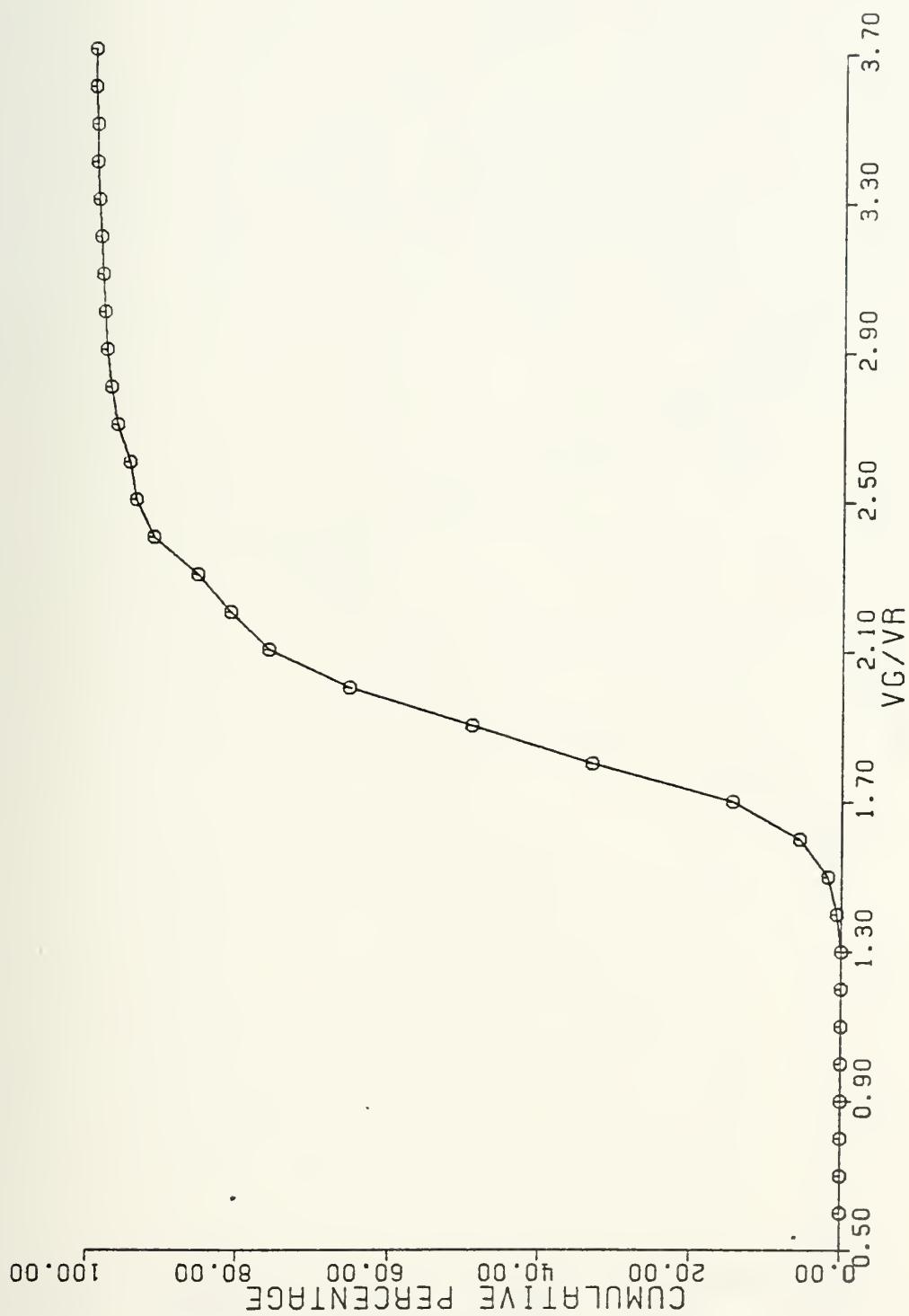


Figure 32B. Cumulative percentage distribution of  $V_G/V_R$  for extreme wave groups: largest group  $V_G/V_R$  per wave record.





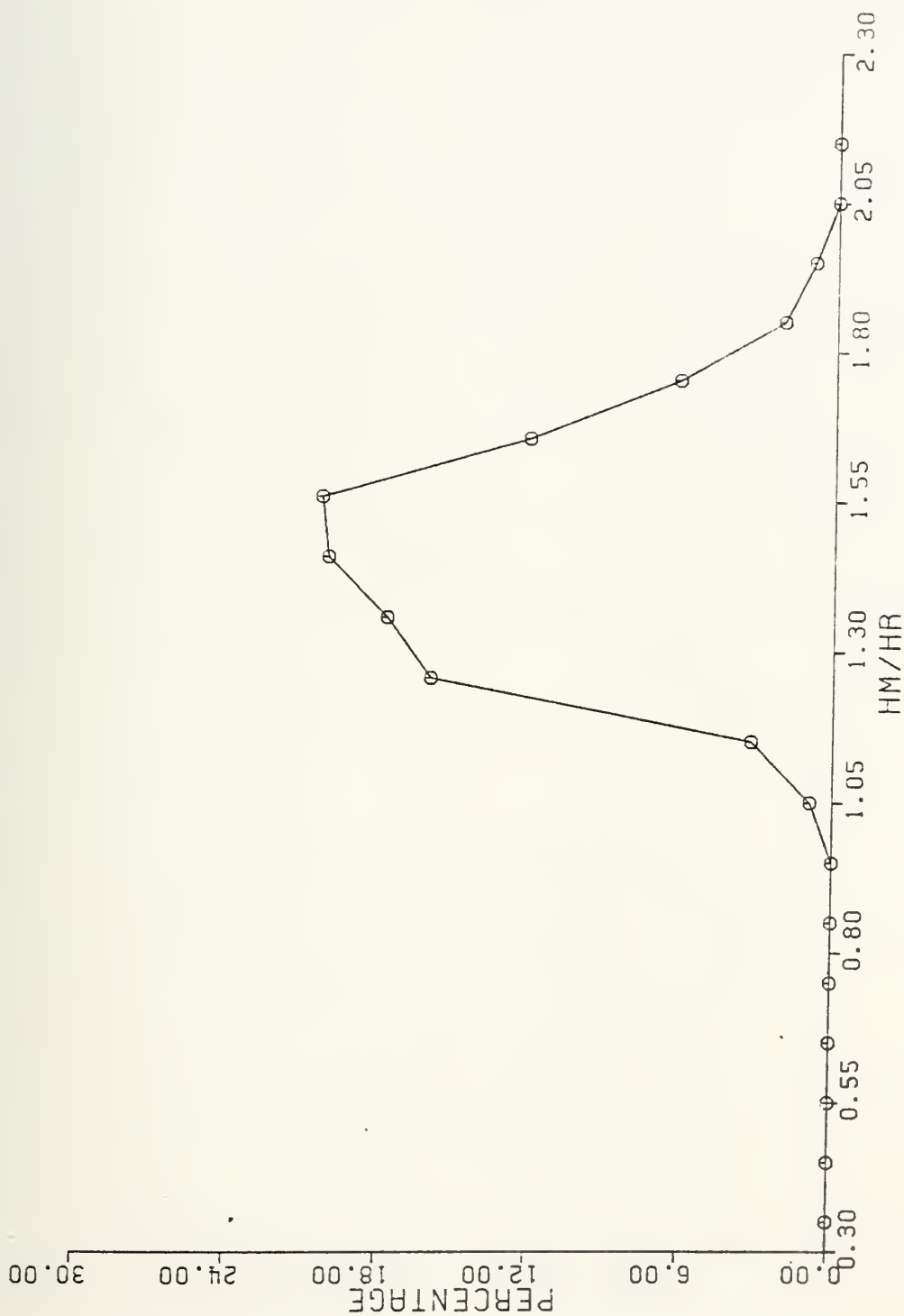


Figure 33A. Histogram of  $H_M/H_R$  for extreme wave groups: largest group  $V_G/V_R$  per wave record.



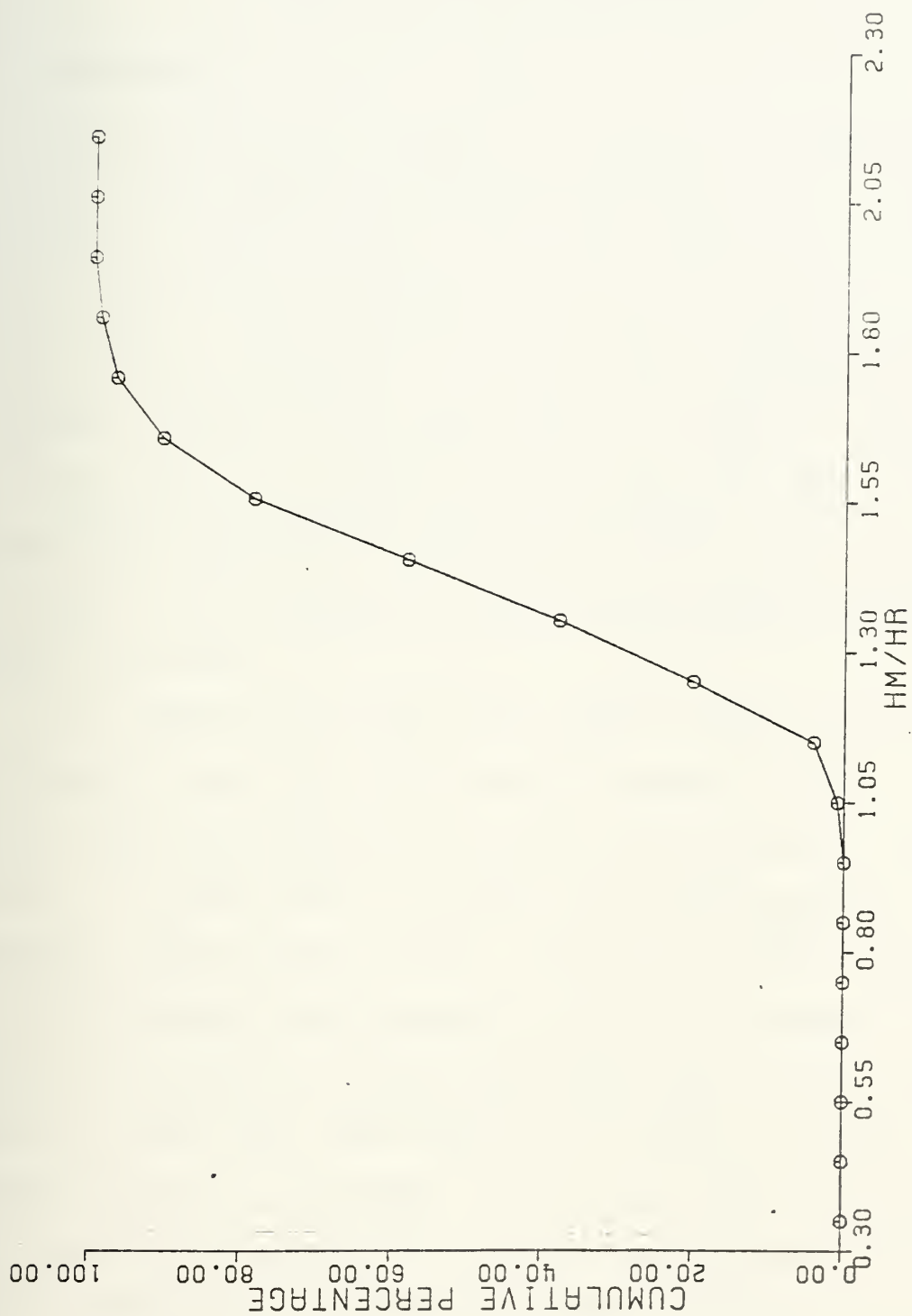


Figure 33B. Cumulative percentage distribution of  $H_M/H_R$  for extreme wave groups: largest group  $V_G/V_R$  per wave record.



## APPENDIX:

### WAVE RECORD SPECTRAL AND WAVE GROUP STATISTICAL RELATIONS

#### A. BACKGROUND

The initial objective of this study was to examine statistical relationships among wave group and wave record parameters, and the results have been presented in the main text. As the study progressed, however, it became evident that it might be possible to relate group statistics to the frequency spectra of the wave records analyzed. Time was a critical factor and the analytical methods employed were simple, but the results were very promising and have been included in this appendix. The need for a more in-depth study is evident.

It is generally thought that spectra that are unimodal or bimodal (one or two main spectral peaks) indicate wave trains that have traveled from one or two sources, respectively. In the study of unimodal wave record spectra, Thompson (1972), Smith (1974), and Sedivy (1978) have shown that the spectral peak occurs at a certain frequency or period in a spectrum because the wave groups contain quasi-periodic waves that approximate this period of maximum energy density. This seems logical since wave groups may be thought of as packets of energy concentration. In other words, wave groups with the most energy compared to the energy contained in the wave record as a whole ( $V_G/V_R$ )



highest) should be responsible for that period marking the spectral peak. It also seems reasonable to expect that aside from the main peak of the spectrum, wave groups might possibly be responsible for subsidiary maxima, including bumps, peaks, and other irregularities, that commonly are observed in frequency spectra.

Limited time permitted only a preliminary examination of these suppositions, and the results obtained are presented below. The results are promising but far from conclusive. More research is needed in this area for definitive conclusions to be drawn.

#### B. RELATIONSHIP BETWEEN GROUP STATISTICS AND THE FREQUENCY SPECTRUM

The first area examined was the relationship of average wave group periods and their respective  $V_G/V_R$  ratios in a given record to the frequency spectrum of the record. Ten wave records were examined for this relationship. The frequency spectra were printed out and the  $T_G$  and  $V_G/V_R$  value for each group plotted thereon. Figure A-1 shows an example. As expected, most of the wave groups are seen to fall in the area of the spectral peak although not necessarily on the peak itself. An interesting feature of the figure is that not a single group falls to the left of the peak. Also noteworthy is the fact that those groups with high values of relative energy do not necessarily fall closest to the spectral peak period, nor do groups of low relative energy





fall in the tails of the curve. The other feature that deserves attention is that some of the more prominent secondary peaks show no wave group falling near their peak periods. It is notable that without exception every wave record so examined showed similar results.

In an attempt to determine why the plotted wave groups do not bracket the frequency peak as logic would dictate, four of the digitized wave records were printed out in analog form to large scale and hand analyzed. The first important observation that was immediately apparent concerned the accuracy to which the average wave group period can be calculated. The computer, which was constrained by the one-second digitizing rate of the data collection system, was accordingly limited to calculating wave periods to the nearest second. The hand analysis method could use interpolation and determine wave periods to a tenth of a second. Since the average group period is an average of the periods of the individual waves in the group, it was possible to determine  $T_G$  to an accuracy an order of magnitude better than the computer analysis. With the accuracy of  $T_G$  thus increased the spectra and group statistics were replotted as shown in Figure A-2. The same problems as before were still apparent. The positions of the wave groups had all shifted slightly to the left but their relationship to the spectral peak remained unchanged.

A problem area that became readily apparent was that most wave groups contain one or more waves of smaller amplitude,



and almost without exception also of shorter period, relative to the other waves in the group. Figures A-3 and A-4 show groups with waves radically different from the other waves in the group. As wave groups can be thought of in terms of not only energy content but also in terms of regularity of both the height and period of the successive waves that comprise the groups, the question arose as to whether anomalous waves should be included in a group for statistical purposes.

This particular problem arose from using the zero up-crossing method that identifies waves solely on the basis of the waveform rising (however slightly) above the mean water level. Relatively small or insignificant waves would not be counted if wave groups were being observed visually. In fact, at times, they might not even be seen against the random wave field background. It was therefore apparent that another restriction needed to be added to the wave group definition. A disqualification factor based on either the height or the period of the subject wave would be logical.

Using wave period as the basis for disqualification, a wave deletion parameter was applied under the following conditions. If a group contains only two waves neither of the two waves would be disqualified. If the group has three or four waves the shortest period wave, if any, would be disqualified. If the group contains five or more waves the two shortest period waves would be disqualified. Using the



more accurate hand-analyzed data, this method of wave disqualification was applied and  $T_G$  was calculated and was plotted as before on the power spectrum. The results are illustrated in Figure A-5 for the same wave record as shown in Figures A-1 and A-2. The results are startlingly different. All wave groups now fall close to and on either side of the main spectral peak. The results were similar for the other wave records so analyzed. The  $V_G/V_R$  values, however, again displayed no tendency toward being larger as  $T_G/T_R$  approaches one. No wave groups were found to fall on or near secondary maxima; however, later analysis showed that those waves occurring in the intervals between wave groups when systematically analyzed fall in the tails of the spectrum. These results show that a relationship evidently exists between the quasi-periodic waves constituting a group and the peak period of the system, but that the methods and definitions used in the machine analysis were too coarse to spell out this relationship.

It should be noted that the wave disqualification scheme described above and used in the subsequent analyses represented a first attempt in the interest of time. This is a very crude method for wave deletion and more sophisticated methods (such as filtering) or definitions would probably yield better results.



### C. RELATIONSHIP BETWEEN INDIVIDUAL WAVES AND THE FREQUENCY SPECTRUM

As mentioned previously, the energy represented in a wave record may be thought of as proportional to the squares of the heights of the frequency components that are summed linearly to give the observable waves on the sea surface. However, if the wave record is considered to be composed of a finite number of components and if each wave in the record can be assumed to be representative of one of these components, then the energy of the wave record may be expected to be proportional to the squares of the heights of the waves in the wave record. Data from the hand analysis of wave records accomplished previously were coupled with this assumption and the results used to construct a form of an energy density curve. If these pseudo-energy curves thus constructed are similar to the frequency spectra for the same wave records, then a statement could be made relating individual waves in a wave record to the frequency spectrum.

The first analysis approach was made by identifying each successive wave in a record as belonging to a wave group or to the interval between wave groups. The wave groups were also identified by the standard  $V_G/V_R$  bands specified earlier. The heights of the individual waves were squared and plotted against the respective frequency. Figure A-6 illustrates this analysis for the same wave record shown in Figure A-5 (and several earlier figures). This figure is remarkably





similar in appearance to the actual spectral density curve for the same wave record. This is true with regard to both the distribution and magnitude of the energy. Waves belonging to wave groups fall within a relatively narrow frequency band while waves belonging to intervals between groups, termed interval waves, are uniformly distributed over the full range of frequencies found in the spectrum. As expected, the interval waves are of low energy content while most of the group waves are high in energy. This is shown by the gradation from interval to group waves from the bottom to the top of the graph. The other notable feature of this figure is that the high energy waves that comprise the higher values around the spectral peak are waves from groups with a high  $V_G/V_R$  ratio.

Because this type of analysis is clearly suggestive of a spectral energy distribution, another method was employed to generate a pseudo-spectrum. This involved summing the squares of the height values and cumulating them over the full frequency range. Differential values, corresponding to the width of the frequency band desired, were read from the cumulative curve and represented the amount of energy in these bands. Values were then plotted at the midpoint of each band and a continuous pseudo-spectral curve generated. The results of this method are shown in Figure A-7. The plotted values were obtained from the cumulative curve using



a frequency bandwidth of 0.01 Hz. Curves for bandwidths other than 0.01 Hz were generated, including a bandwidth of 0.0078 Hz which was used in the computer program to generate the actual frequency spectrum. The bandwidth of 0.01 Hz was used for several reasons. Shorter bandwidths tended to give too much irregularity in the pseudo-spectral curve while longer bandwidths tended to smooth secondary features out entirely. Also, this choice of bandwidth was extremely simple to work with, especially in the initial hand analysis. A comparison between the actual spectrum and the pseudo-spectrum shows that the locations of the main and secondary peaks are in close agreement with those of the frequency distribution. The relative heights between peaks do not closely agree, however.

#### D. SUMMARY

A preliminary analysis of the relationships of wave groups and individual waves (both within and without wave groups) with the energy density spectrum was performed as a natural outgrowth of the wave group statistical analysis. Digital determination of the group period from the wave records contained accuracy limitations related to the digital sampling rate, and anomalous waves within groups (both in period and height) were shown to be a possible source of deviation of the average group period from the spectral peak period. This may explain why  $T_G/T_R$  values in this study tend to be



less than unity and why the  $T_G/T_R$  distributions found are not in close agreement with the results of manual analyses of wave groups from previous studies. Indications that groups of high relative energy are responsible for the period of the spectral peak were shown, and it was also demonstrated that individual waves of large amplitude (and energy) contained in these high energy groups appear to be the factor controlling the spectral peak period. Also shown is the generation of a pseudo-spectrum from a simple statistical analysis of individual wave heights that is very similar in most respects to the actual energy spectrum.



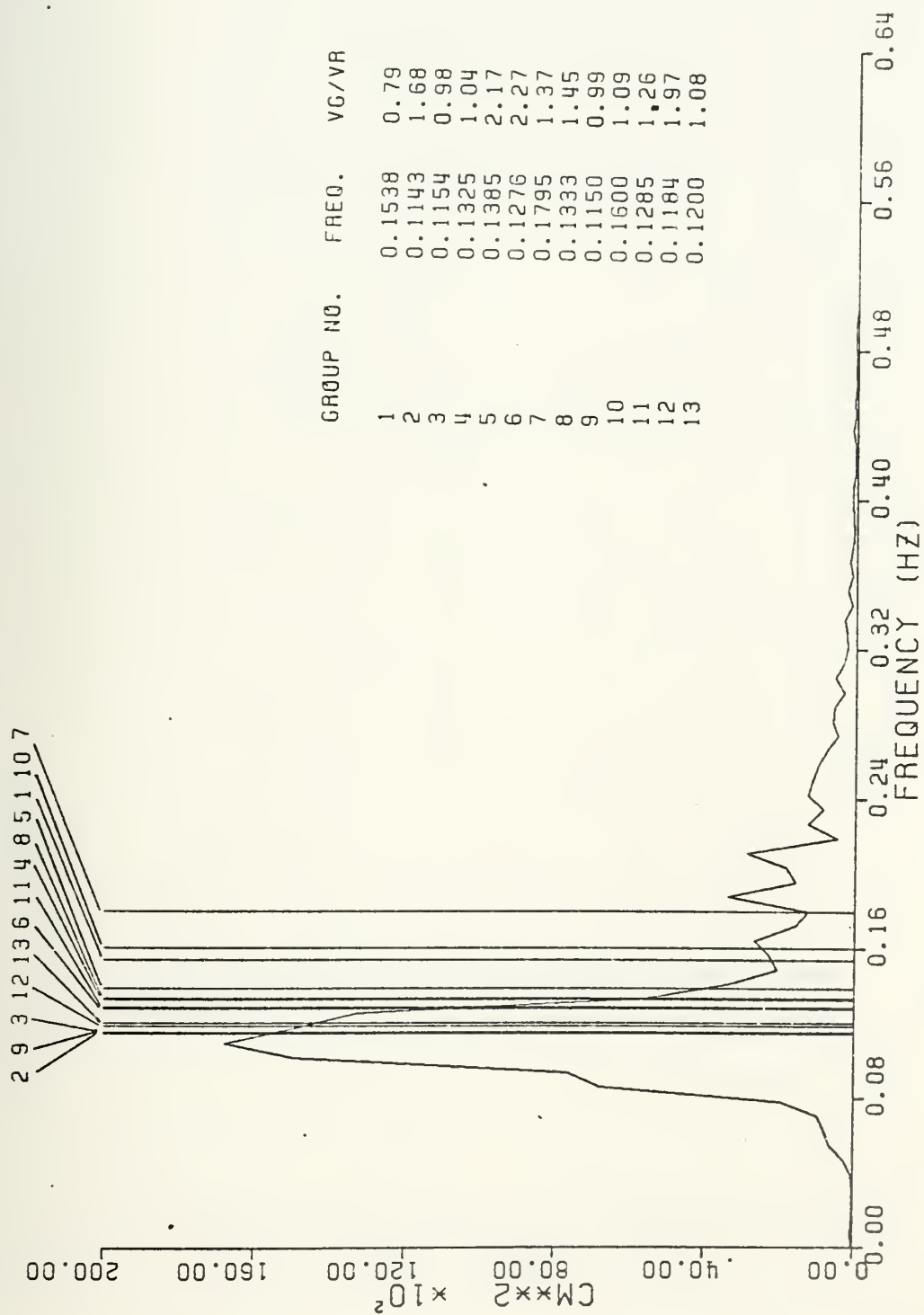


Figure A-1. Machine analysis of group  $T_G$  and  $V_G/V_R$  for wave record number 143 plotted on corresponding frequency spectrum.





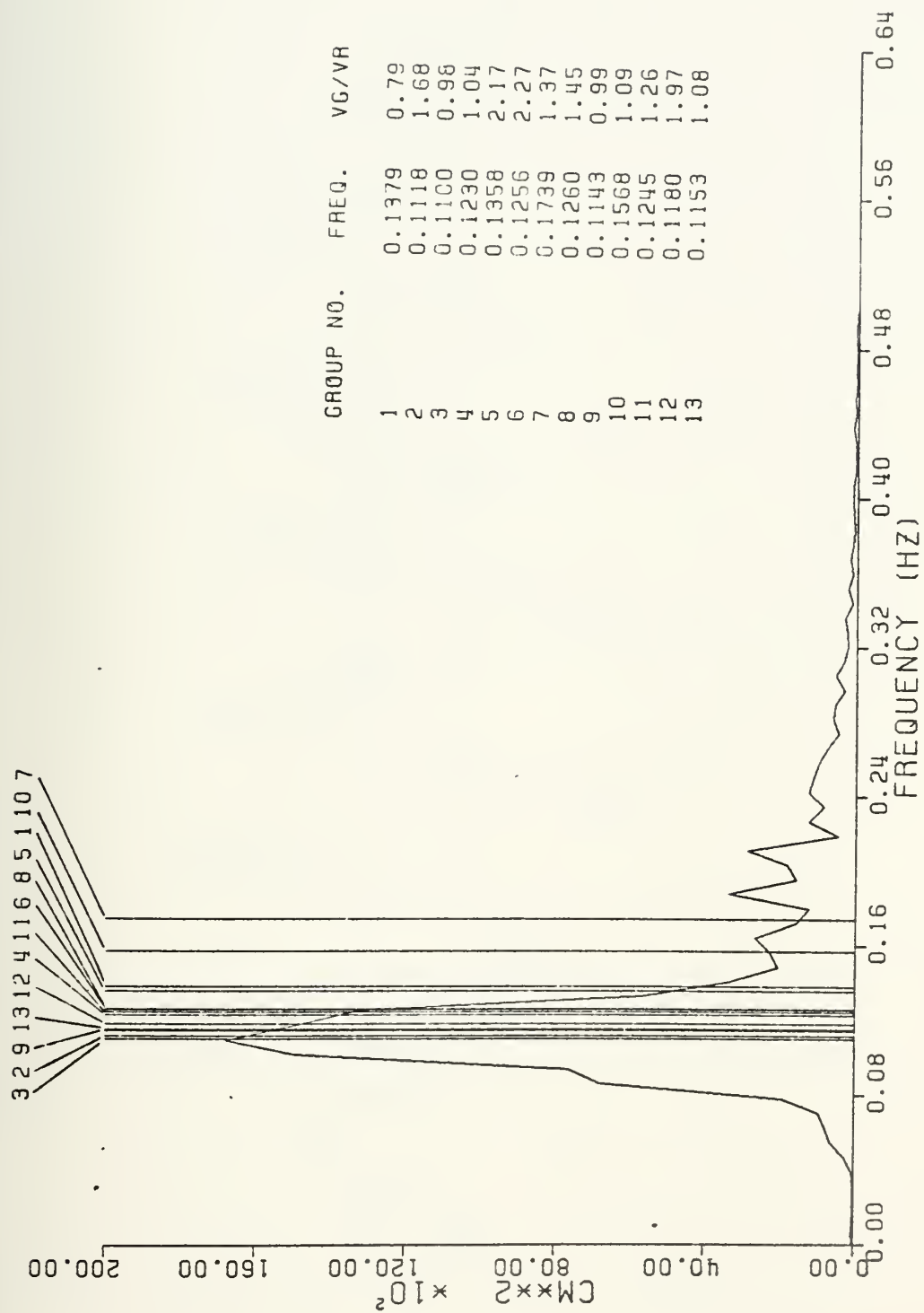


Figure A-2, Hand analysis of group  $T_G$  and  $V_G/V_R$  for wave record number 143 plotted on corresponding frequency spectrum.



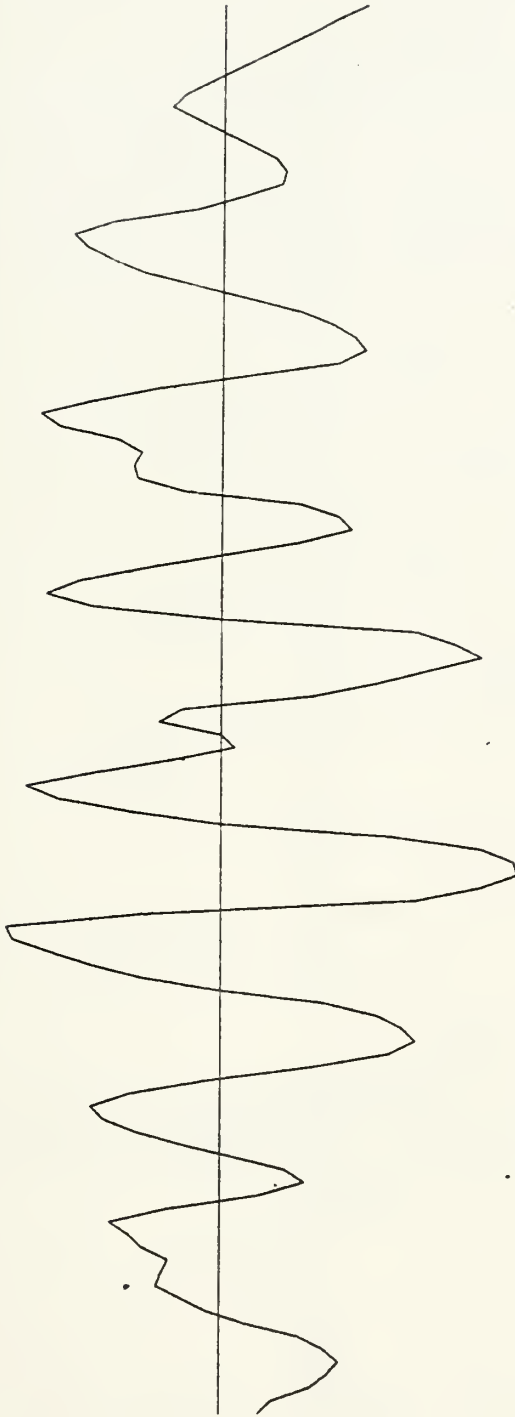


Figure A-3. Example of wave group containing non-periodic anomalous wave in middle of wave group.



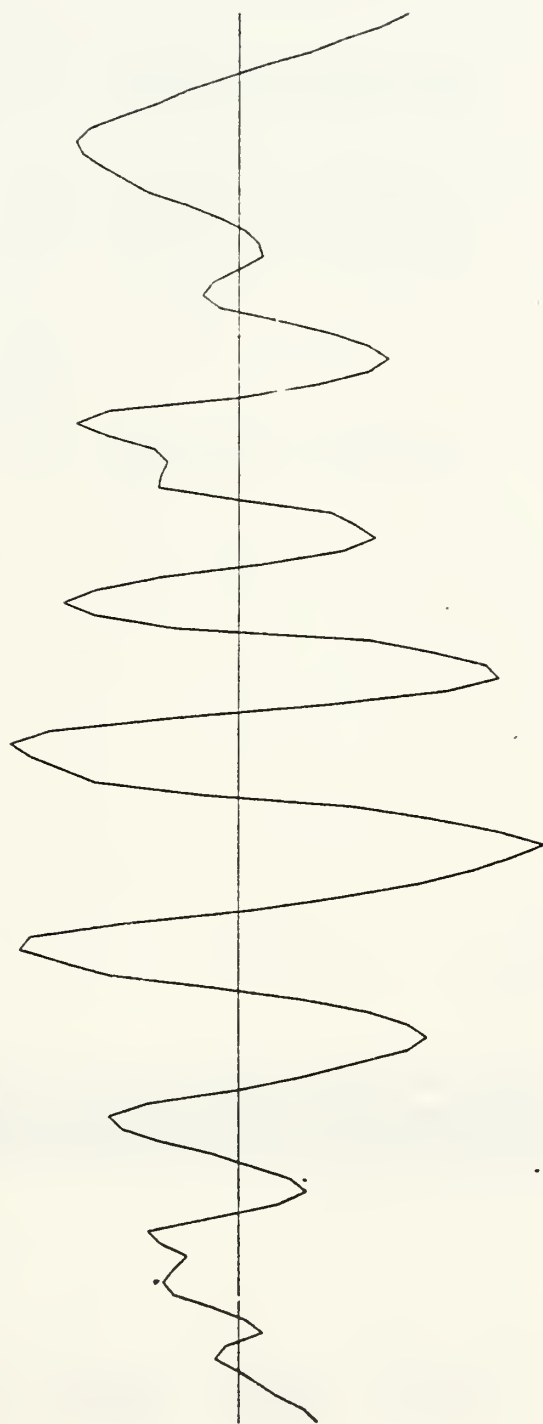


Figure A-4. Example of wave group containing non-periodic anomalous waves at beginning and end of wave group.



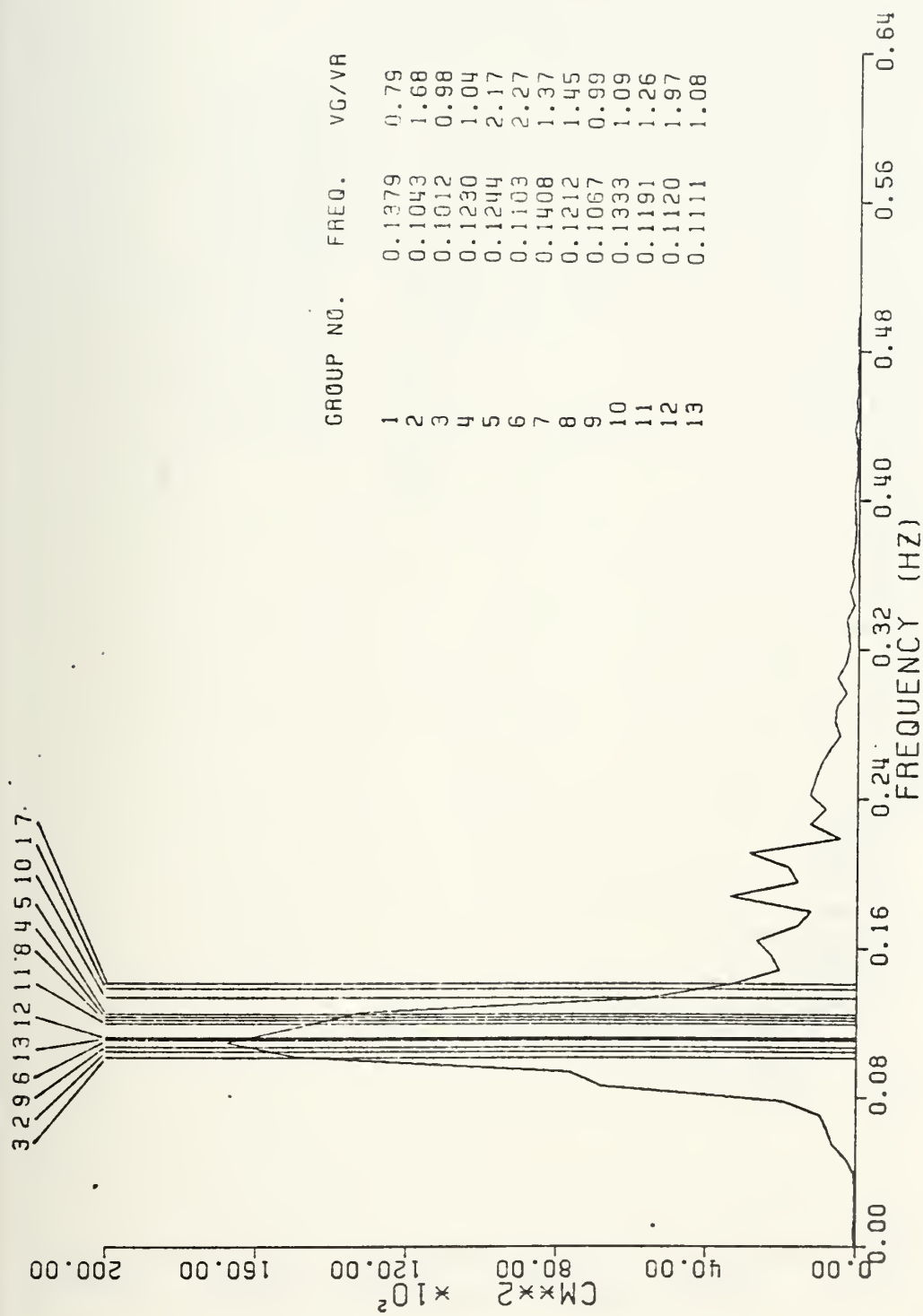


Figure A-5. Improved (anomalous wave deletion) hand analysis of group T<sub>6</sub> and V<sub>G</sub>/V<sub>R</sub> for wave record number 143 plotted on corresponding frequency spectrum.





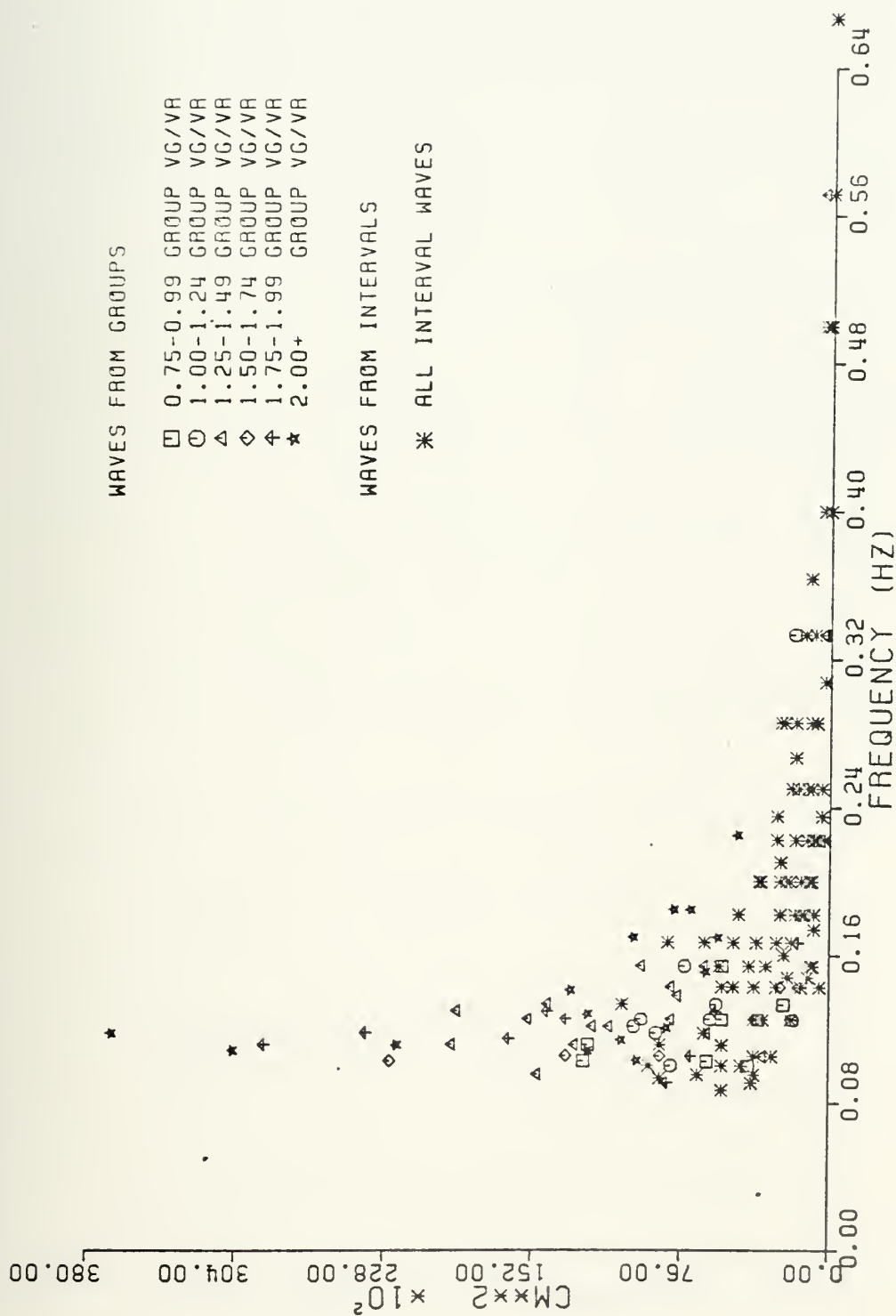


Figure A-6. Plot of squares of individual wave heights versus frequency for wave record number 143.



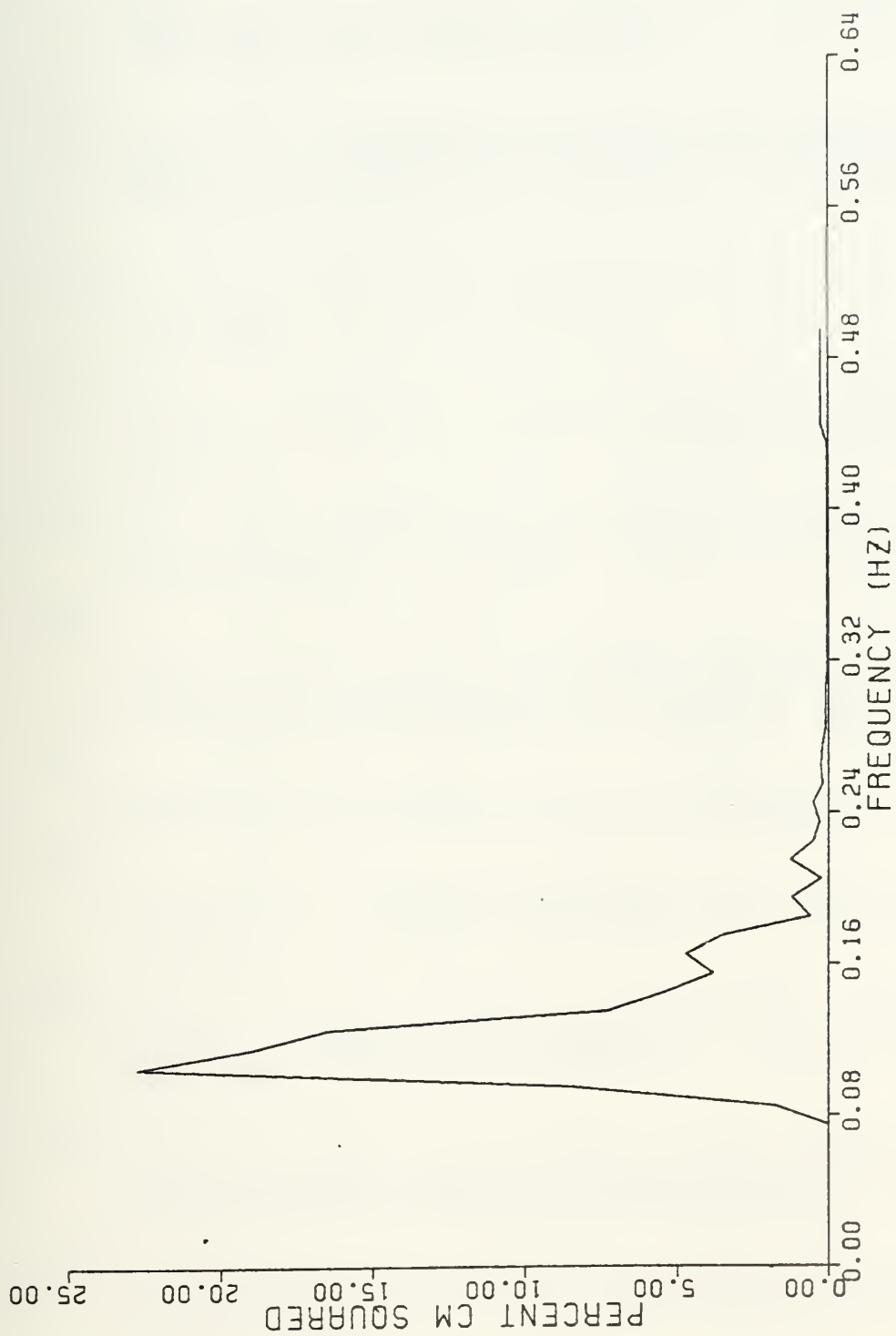


Figure A-7. Plot of pseudo-spectrum for wave record number 143 generated by cumulating squared wave heights.



## LIST OF REFERENCES

1. Chakrabarti, S. K., and R. P. Cooley, 1977, "Statistical Distribution of Periods and Heights of Ocean Waves", Journal of Geophysical Research, v. 82 no. 9, p. 1363-1368.
2. Chou, S., 1978, Selection of Optimum Wave Conditions for Time-Domain Simulation of Non-Linear Phenomena, Doctor of Engineering Dissertation, University of California, Berkeley.
3. Ewing, J. A., 1973, "Mean Length of Runs of High Waves", Journal of Geophysical Research, v. 78 no. 12, p. 1933-1936.
4. Forristall, G. Z., 1978, "On the Statistical Distribution of Wave Heights in a Storm", Journal of Geophysical Research, v. 83 no. C5, p. 2353-2358.
5. Goda, Y., 1970, "Numerical Experiments on Wave Statistics with Spectral Simulation", Report of the Port and Harbor Research Institute, v. 9 no. 3, p. 3-57.
6. Goda, Y., 1974, "Estimation of Wave Statistics from Spectral Information", American Society of Civil Engineers, Proceedings of the International Symposium on Ocean Wave Measurement and Analysis, p. 320-337.
7. Goda, Y., 1976, "On Wave Groups", Proceedings of the Behaviour of Offshore Structures, 1976, p. 115-128.
8. Goda, Y., 1979, "A Review on Statistical Interpretation of Wave Data", Report of the Port and Harbor Research Institute, v. 18 no. 1, p. 5-32.
9. Hamilton, J., W. H. Hui, and M. A. Donelan, 1979, "A Statistical Model for Groupiness in Wind Waves", Journal of Geophysical Research, v. 84 no. C8, p. 4875-4884.
10. Harris, D. L., 1970, "The Analysis of Wave Records", American Society of Civil Engineers, Proceedings of the 12th Coastal Engineering Conference, p. 85-100.
11. Longuet-Higgins, M. S., 1952, "On the Statistical Distribution of the Heights of Sea Waves", Journal of Marine Research, v. 11 no. 3, p. 245-266.
12. Michel, W. H., 1968, "Sea Spectra Simplified", Marine Technology, v. 5 no. 1, p. 17-30.



13. Mollo-Christensen, E., and A. Ramamonjiarisoa, 1978, "Modeling the Presence of Wave Groups in a Random Wave Field", Journal of Geophysical Research, v. 83 no. C8, p. 4117-4122.
14. Rye, H., 1974, "Wave Group Formation Among Storm Waves", American Society of Civil Engineers, Proceedings of the 14th Coastal Engineering Conference, p. 164-183.
15. Sedivy, D. G., 1978, Ocean Wave Group Analysis, M.S. Thesis, Naval Postgraduate School, Monterey,
16. Siefert, W., 1976, "Consecutive High Waves in Coastal Waters", American Society of Civil Engineers, Proceedings of the 15th Coastal Engineering Conference, p. 171-182.
17. Smith, R. C., 1974, Ocean Swell Wave Groups from Wave Record Analysis, M.S. Thesis, Naval Postgraduate School, Monterey.
18. Thompson, W. C., 1970, "Swell and Storm Characteristics from Coastal Wave Records", American Society of Civil Engineers, Proceedings of the 12th Coastal Engineering Conference, p. 33-51.
19. Thompson, W. C., 1972, "Period by the Wave Group Method", American Society of Civil Engineers, Proceedings of the 13th Coastal Engineering Conference, p. 197-214.
20. Thompson, W. C., and F. M. Reynolds, 1976, "Ocean Wave Statistics from FNWC Spectral Analysis", American Society of Civil Engineers, Proceedings of the 15th Coastal Engineering Conference, p. 238-257.
21. Thompson, W. C., and R. C. Smith, 1974, "Wave Groups in Ocean Swell", American Society of Civil Engineers, Proceedings of the International Symposium on Ocean Wave Measurement and Analysis, p. 338-351.
22. Wilson, J. R., and W. F. Baird, 1972, "A Discussion of Some Measured Wave Data", American Society of Civil Engineers, Proceedings of the 13th Coastal Conference, p. 113-130.





# INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2. Library, Code 0142 Naval Postgraduate School Monterey, California 93940	2
3. Chairman Department of Oceanography, Code 68 Naval Postgraduate School Monterey, California 93940	1
4. Chairman Department of Meteorology, Code 63Ha Naval Postgraduate School Monterey, California 93940	1
5. Professor Warren C. Thompson Department of Oceanography, Code 68Th Naval Postgraduate School Monterey, California 93940	5
6. Lieutenant A. R. Nelson Officer-in-Charge Naval Oceanography Command Detachment Fleet Activities Okinawa FPO Seattle, Washington 98770	3
7. Department of Oceanography, Code 68 Naval Postgraduate School Monterey, California 93940	3
8. Library, Code 63 Department of Meteorology Naval Postgraduate School Monterey, California 93940	1
9. Director Naval Oceanography Division Navy Department Washington, D.C. 20350	1
10. Commander Naval Oceanography Command NSTL Station Bay St. Louis, Mississippi 39529	1



11. Commanding Officer |  
Fleet Numerical Oceanography Center  
Monterey, California 93940
12. Commanding Officer |  
Naval Environmental Prediction  
Research Facility  
Monterey, California 93940
13. Commanding Officer |  
Naval Ocean Research and Development Activity  
NSTL Station  
Bay St. Louis, Mississippi 39529
14. Office of Naval Research (Code 480) |  
Naval Ocean Research and Development Activity  
NSTL Station  
Bay St. Louis, Mississippi 39529
15. Scientific Liaison Office |  
Office of Naval Research  
Scripps Institution of Oceanography  
La Jolla, California 92037
16. Library |  
Scripps Institution of Oceanography  
P.O. Box 2367  
La Jolla, California 92037
17. Library |  
Department of Oceanography  
University of Washington  
Seattle, Washington 98105
18. Chief, Ocean Services Division |  
National Oceanic and Atmospheric Administration  
8060 Thirteenth Street  
Silver Spring, Maryland 20910
19. Library |  
Department of Oceanography  
Oregon State University  
Corvallis, Oregon 97331
20. Commander |  
Oceanographic Systems Pacific  
Box 1390  
Pearl Harbor, Hawaii 96860
21. Library |  
Naval Ship Research and Development Center  
Bethesda, Maryland 20084



22. Dr. Richard J. Seymour |  
California Department of Navigation and  
Ocean Development  
Mail Code A-022  
Scripps Institution of Oceanography  
La Jolla, California 92037
23. Mr. Bill Satow |  
Department of Navigation and Ocean  
Development  
State of California  
1416 Ninth Street  
Sacramento, California 95814
24. Library |  
Naval Coastal Systems Laboratory  
Panama City, Florida 32401
25. Library |  
Naval Civil Engineering Laboratory  
Port Hueneme, California 93043
26. Library |  
Coastal Engineering Research Center  
Kingman Building  
Fort Belvoir, Virginia 22060
27. Library |  
Waterways Experiment Station  
U.S. Army Corps of Engineers  
P.O. Box 631  
Vicksburg, Mississippi 39180
28. Director |  
National Ocean Survey  
Rockville, Maryland 20852
29. Mr. Orville T. Magoon |  
Coastal Engineering Branch  
Corps of Engineers, South Pacific Division  
630 Sansome St., Room 1216  
San Francisco, California 94111
30. Dr. D. Lee Harris |  
Waterways Experiment Station  
U.S. Army Corps of Engineers  
P.O. Box 631  
Vicksburg, Mississippi 39180



24 DEC 80  
24 SEP 82

S99.49  
S12809

Thesis  
N3579 Nelson  
c.1 Statistical prop-  
erties of ocean wave  
groups.

187042

24 DEC 80  
24 SEP 82

S99.49  
S12809

Thesis  
N3579 Nelson  
c.1 Statistical prop-  
erties of ocean wave  
groups.

187042

thesN3579

Statistical properties of ocean wave gro



3 2768 002 01796 4

DUDLEY KNOX LIBRARY